University of Miskolc

Faculty of Mechanical Engineering

Numerical Analysis of Mechanical Contact

Theses of the Ph.D. Dissertation

written by

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1. Brief Summary of the Dissertation

The p-extension of the finite element method can be well applicable for solving contact problems in linear elasticity with high accuracy. The discretization of the contact domain with these elements is advantageous since it results in fast convergence and the high order mapping assures accurate geometry for shape optimization.

In the case of problems, where the resulting contact domain is much smaller than the bodies in contact, the solution can be achieved by using the formulae based on the elastic half space theory.

In the thesis two different contact tasks are examined with the use of finite element method.

1. In connection with shape optimization of punch (axially symmetric) problems several new contact optimization tasks are presented with an additional condition for taking the ultimate stress of the real material into account.

2. For three dimensional contact problems a new border positioning technique is introduced to ensure that the edges of elements coincide with the border of the contact domain.

The new contact optimizations have a stress based condition. To take the stress condition into account during shape optimization a new iterative procedure – named as 2^{nd}-*type iteration* – is introduced which is based on a simple linearization.

To carry out axially symmetric optimizations the shape of the contacting particles are modified by a control function which controls the contact pressure distribution.

The shape optimization of non-centrally loaded rollers are also investigated. The peak values in stresses at the ends of the rollers can be minimized by controlling the contact pressure. The effect of friction is also examined in the case of centrally loaded rolling elements. However, the conclusion is that the influence of friction is not significant for the optimal shape.

Numerical experiments show that the use of high order p-extension finite elements results in oscillation near singular points or edges. In the case of contact problems the calculations can be achieved more precisely by using finite elements which are aligned along the border of contact and separation zones. It means that the numerical procedure ensures that an element is either in full contact, or non-contacting at all, i.e. the initial *C-type* problem is transformed into *B-type* task.

To describe the contact border in three dimensions a spline curve is used. The application of spline technique ensures high effectiveness and a new modelling method for solving problems in contact mechanics. A general solution for three dimensional contact problems using spline positioning is not known in scientific literature so far.

A computational program is developed for solving shape optimizations in two dimensions and for handling contact problems with p-extension elements in three dimensions. The proposed algorithms are based on iterational techniques in which the elements are positioned automatically i.e. adaptively in order to achieve more precise results.

The numerical experiments, both in two and three dimensions, prove that highly accurate solutions can only be given with p-extension finite elements if the calculations use only the elements which are transformed to the contact border.

2. New scientific results

The new scientific results of the dissertation are summarized by the following theses

1. By controlling the contact pressure new contact optimization tasks are defined in the case of the contact of axially symmetric bodies which allow for considering the stress constraint of the material. The optimized quantities are llisted in Table 1.

Table 1 Quantities to be optimized

w_0	maximization of the prescribed displacement
F_p	maximization of the compression force
M_T	maximization of torque
D	minimization of frictional power lost

2. For the solution of the optimization tasks defined in thesis 1 a new iterational technique is developed. The introduced method consists of two different sequential iterations. The 1^{st}-*type* iteration performs shape optimization with fixed control parameters; while the 2^{nd}-*type* iteration fulfills the condition for the stress constraints, which operates mainly in a linear way. The convergence of the proposed iterational technique, which solves a non-linear optimization, is proved by numerical experiments resulting from finite element calculations.

3. The shape optimization of rolling elements is achieved on the basis of the elastic half space theory.

   (a) When friction is not taken into account, by controlling the contact pressure distribution the shape of a rolling element is optimized by a modified control function. The optimization process is illustrated by numerical experiments.

(b) The shape of a rolling element is optimized by pressure control in which the effect of friction is also taken into account. Numerical calculations demonstrate the results of the proposed algorithm.

4. Handling three dimensional contact problems in a new numerical way.

   (a) The border of the resulting single connected contact domain is limited by a parametric curve which can be a closed B-spline or NURBS curve. To ensure the interpolation through the contact separation points, a linear algebraic equation system is set and must be solved. A computational program is developed to illustrate interpolation numerically.

   (b) To position the contact elements along the resulting contact border, a new iterational technique is introduced. The edges of the elements are aligned to the contact separation zone in an adaptive way which ensures accurate convergence in the solution. Henceforward the application of the p-extension finite elements in three dimensions with elements' positioning produces exponential convergence.

   (c) Using three dimensional p-extension finite elements a new – previously unknown in scientific literature – algorithm is developed which is coded in Fortran 90. On the basis of the solution of the contact problem, which is resolved by a penalty method, the program operates in the following way:

      – the interpolation points are searched along a prescribed direction with linear approximation,
      – an interpolation spline is generated,
      – the parameters for the approximation of the finite elements are produced on the basis of least square method,

– the newly formed finite elements and the mesh are generated together with the element stiffness and load matrices,

then the contact problem is solved on the basis of the steps (algorithm) used in previous finite elements programs.

3. Publications in the Field of the Dissertation

Scientific Articles in Periodicals written in English:

⟨1⟩ Páczelt, I. & Baksa, A. (2002). Examination of contact optimization and wearing problems. *Journal of Computational and Applied Mechanics*, 3(**1**), pp. 61-84.

Scientific Articles in Periodicals written in Hungarian:

⟨2⟩ Baksa, A. & Páczelt, I. (2001). Some new contact optimization problems with iteration. *GÉP*, 52(3-4), pp. 38-42.

⟨3⟩ Baksa & Páczelt, I. (2004). Approximation of contact domain with the use of B-splines, *GÉP*, 55(1), pp. 8-13.

⟨4⟩ Baksa, A. (2005). Érintkezési feladat megoldása háromdimenziós *p*-verziójú végeselemek segítségével. *GÉP*, 56(5), pp. 1-12.

Scienfic Reviews in Conference Brochures written in English:

⟨5⟩ Baksa, A. & Páczelt, I. (2001). Optimization problem of rolling body contacts. 3^{rd} *International Conference of PhD Students*, University of Miskolc, pp. 1-8.

⟨6⟩ Baksa, A & Páczelt, I. (2001). Some new contact optimization problems. *microCAD'2001 International Scientific Conference*, Section N: Applied Mechanical Engineering Sciences, University of Miskolc, pp. 7-12.

⟨7⟩ Páczelt, I. & Baksa, A. (2001). Solution of contact optimization problems with iteration. *ECCM 2001 European Conference on Computational Mechanics*, Cracow, Poland, CD Proceedings.

⟨8⟩ Baksa, A. & Páczelt, I. (2002). Solution of contact problems with FEM and parallel technique. *microCAD'2002 International Scientific Conference*, Section D1: Mechanical Engineering Sciences, University of Miskolc, pp. 1-5.

⟨9⟩ Baksa, A. & Páczelt, I. (2004). Approximation of contact domain with the use of B-splines. *microCAD 2004, International Scientific Conference*, Section E: Applied Mechanics, University of Miskolc, pp. 7-12.

⟨10⟩ Páczelt, I. & Baksa, A. (2005). Solution of contact problems with high rate convergence. 8^{th} *U.S. National Congress on Computational Mechanics*, Austin, Texas, July 24-28. Minisympozia: Computational Contact Mechanics, Conference CD Proceedings.

Scientific Presentations in English:

⟨11⟩ Baksa, A. & Páczelt, I. (2002). Solution of contact problems using parallel technique. *nmcm2002, Numerical Methods and Computational Mechanics*, University of Miskolc.

⟨12⟩ Baksa, A. & Páczelt, I. (2002). Different solution methods for contact optimization problems. *Solmech2002, 34th Solid Mechanics Conference*, Zakopane, Poland.

Scientific Presentations in Hungarian:

⟨13⟩ Páczelt, I. & Baksa, A. (1999). Programrendszer érintkezési feladatok megoldására, *VIII. Magyar Mechanikai Konferencia*, Miskolci Egyetem.

⟨14⟩ Baksa, A. & Páczelt, I. (2003). Iterációs technikák érintkezési optimalizálási feladatok megoldásához. *OGET2003, XI. Nemzetközi Gépész Találkozó, Számítógépes tervezés és gyártás*, Erdélyi Magyar Társaság, Kolozsvár, pp. 26-30.

⟨15⟩ Baksa, A. & Páczelt, I. (2003). Érintkezési feladatok vizsgálata magasfokú approximáció segítségével. *IX. Magyar Mechanikai Konferencia*, Miskolci Egyetem.

⟨16⟩ Baksa, A. & Páczelt, I. (2005). Térbeli érintkezési feladat vizsgálata. *OGET2005, XIII. Nemzetközi Gépész Találkozó*, Fogaskerékhajtások, Modern Megmunkálások, Erdélyi Magyar Társaság, Szatmárnémeti, pp. 39-42.

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