UNIVERSITY OF MISKOLC

Faculty of Mechanical Engineering

ANALYSIS AND OPTIMIZATION OF FIBRE REINFORCED PLASTIC BEAMS AND CELLULAR PLATES

Theses of the PhD Dissertation

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1. INTRODUCTION

Composite structures utilize the advantages of different structural layers. These have many advantageous properties, which can not be available by other materials, e.g. high strength, low density, corrosion resistance, high bending stiffness, good vibration damping and aesthetic appearance. These advantageous characteristics can be due to special material structure. The fibre-composite consist of the following parts: a basic matrix and a strengthening phase. The correct material content for the required application can be provided by the adequate determination of types, properties and portion of components.

Composites are used in many industries (space-, military-, automotive-, construction-, machine- and chemical industry) due to the above mentioned properties.

The adequate material selection results a composite material which properties much more advantageous compared to other materials. In practical point of view these composites structures are worth to compare to metal structures. Application of composite materials instead of metals provides significant weigh saving due to the low density of composite materials. Weight reduction is a very important property which is utilized in many fields of industry, e.g. automotive-, air and space- and construction industries.

Design procedure, material- and shape optimization of composite structures are much more complex compared to traditional metal structures due to orthotropic or anisotropic nature of composites. Unfortunately simple practical design procedures and calculation methods are not available due to this complexity. It provides a challenge to achieve the aims of my PhD dissertation which are the elaboration of calculation methods for static and dynamic characteristics and behaviour of fibre reinforced composites (FRP), elaboration of optimization method for optimal material-content and structure of defined complex real structures.

2. AIMS OF THE PHD DISSERTATION, METHODS FOR ACHIEVEMENT OF THESE

The topics of the dissertation are: history of development of fibre composites, introduction of components and properties, elaboration of methods for calculation and structural optimisation of single- and multi-cellular structures, determination of weight reduction achieved by application of composite structures instead of metal structures.

2.1. I completed the calculation of maximal middle deflection of pultruded FRP I- and box beams. The mass of a given FRP I- and box beams was compared to steel beams which have same inertia. The result of the comparison is that significant weight reduction can be achieved by application of composite structures (67% in case of FRP I beam, 59% in case of FRP box beam) compared to steel beams for case of same span, loading condition and deflection constraint.

2.2. The new experimental structural model is the combination of a sandwich structure and a cellular plate structure. Sandwich structure is constructed from steel or FRP deck plates and foam or honeycomb core inside plates. The cellular plate is constructed for steel deck plates and steel stiffeners between these. The examined new structure constructed from FRP deck plates riveted to the upper and lower flanges of aluminium (Al) square hollow section (SHS) profiles:

The constructed new model is the combination of sandwich structure and cellular plate. It means that the model is the combination of materials, stiffeners and manufacturing technologies. These kind of sandwich structures can be used in many applications (e.g. ship deck plate, bridge, air plane, building foundation, etc.).

I completed the measurement (completed at the laboratory of our department) and calculation of structure behaviour in case of static loading and compared the results of measurements and calculations. I verified the goodness of the elaborated calculation method.

I also completed also dynamic tests by Brüel & Kjaer measuring device to determine the eigenfrequency and vibration damping factor of the test specimen. The obtained vibration damping factor was compared to the damping factor of a simple *AL* SHS profile. The result of this comparison is that the application of composite deck plates can cause a significant improvement of vibration damping of the structure.

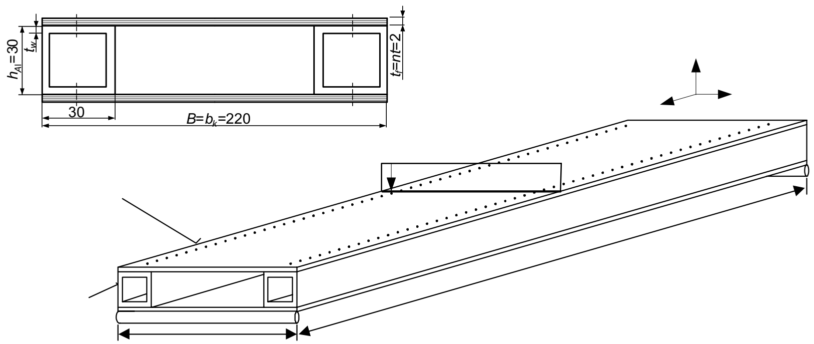

Figure 1. *Sandwich-like structure*

The finite element analysis of the examined structure was completed by the I-DEAS software. Finite element model of the structure was created, obtained results of the analysis relating to deflection and stress distribution were compared to measured and calculated results. These data are near the same, which also verify the goodness of the elaborated calculation method.

2.3. Optimal design of simple- and multi cellular structures was elaborated based on multi-objective functions (mass- and cost functions) and design constraints.

2.3.1. Cost as objective function

Development of an economical construction is a very important design aim due to the high composite material cost. The general cost function can be formulated as the sum of the material and fabrication costs:

$$K = K_m + K_f = k_m \rho V + k_f \sum_{i=1}^{l} T_i \,, \qquad (1)$$

where k_m and k_f are specific material- and fabrication cost factors, ρ is density, V is structure volume, l is number of manufacturing phases, T_i are average times of i^{th} manufacturing phase.

Material-, manufacturing- and assembly costs of all components should be examined in case of complex structures. In our case the total cost is the sum of material costs of composite plates and stiffeners, manufacturing cost of composite laminates and cutting cost of *Al* stiffeners and assembly cost of the total structure:

$$K = K_{dp} + K_s + K_{heatt} + K_f \,. \qquad (2)$$

- Cost of composite deck plates (K_{dp}) is the sum of material costs of two plates (upper, lower).
- Material cost of stiffeners (K_s) is depending on the number of stiffeners, dimension of cross section (h_{Al}, t_w), material density (ρ_{Al}=2,7·10^{-6}

kg/mm[3]), length of stiffeners (L) and specific material cost factors (k_{Al}=4,94 €/kg).

- Cost of heat treatment (K_{heatt}) is depending on the dimension of deck plates and characteristics of applied resin material.
- Fabrication cost (K_f) includes the manufacturing cost of deck plates, cutting of *Al* stiffeners and assembly cost of the total structure. These cost components can be defined as the sum of the times [minutes (min)] required for the manufacturing phases:

$$K_f = k_f \sum_{i=1}^{l} T_i \ , \tag{3}$$

$$\sum_{i=1}^{l} T_i = T_c + T_s + T_{ass} \ , \tag{4}$$

where k_f is the specific fabrication cost which is depending on the stage of development of the given country (k_f=0,5-2 €/min).

Manufacturing time of two composite deck plates (T_c) is depending on the number of layers of laminate (n) and includes the times of form preparation, layer cutting, lamination of layers, heat treatment and finishing works.

$$T_c = T_{pref} + 2[T_{cut} + T_{lam} + T_{postf}] \ ,$$

$$T_c = 10_{\min} + 2[n \cdot 4_{\min} + (n \cdot 2_{\min} + 30_{\min}) + 10_{\min}] \ . \tag{5}$$

Time of manufacturing of stiffeners (T_s) is depending on number of stiffeners (n_s):

$$T_s = n_s \cdot 6_{\min} \ . \tag{6}$$

Time of assembly of the total structure includes the times required for riveting of the structural components:

$$T_{ass} = n_s \cdot 10_{\min} \ . \tag{7}$$

<u>The total manufacturing time can be calculated as the sum of the above mentioned time components:</u>

$$T_{total} = n \cdot 12_{\min} + n_s \cdot 16_{\min} + 90_{\min} \ . \tag{8}$$

The K cost function can be formulated as the sum of the material and fabrication costs

$$K = K_{dp} + K_s + K_{heatt} + K_f$$

$$K(€) = K_{dp} + k_{Al} \ [n_b \, (\rho_{Al} \, 4 \, h_{Al} \ t_w \, L)] + K_{heatt} + k_f \, [\, n \cdot 12_{\min} + n_b \cdot 16_{\min} + 90_{\min}] \ . \tag{9}$$

2.3.2. <u>Mass as objective function</u>

The most advantageous characteristics of the composite structures is the mass saving compared to the traditional steel structures, so to define a minimal weight structure can be a next design aim.

The total mass of the structure (Figure 1, 2) is the sum of the mass of the *Al* and *CFRP* plate components.

$$m = 2 \, \rho_c \, [B \, L(\, n \, t)] + \ n_s \, \rho_{Al} \, [L \, (4 \, h_{Al} \, t_w - 4 \, t_w^{\,2})] \ . \tag{10}$$

2.3.3. <u>Constraints</u>

<u>(1) Deflection of the total structure</u>

The maximal deflection of the total structure is the sum of the calculated deflection and the deflection due to relative movement between the components:

$$w_{\max} = \frac{F \ L^3}{48(E_c I_c + E_{AL} n_s I_{AL})} + \frac{\Delta M \ L^2}{12(E_c I_c + E_{AL} n_s I_{AL})} \le \frac{L}{200} \ . \tag{11}$$

Indexes: c – composite, *Al* – aluminium (stiffener), *s* – stiffener. E_c reduced modulus of the *CFRP* plate, E_{Al} Young's modulus of the *Al* profile, I_c moment of inertia of the *CFRP* plate, I_{Al} moment of inertia of the *Al* stiffener. Due to the stress difference in stress ($\Delta\sigma = |\sigma_{Al} - \sigma_c|$) there is a corresponding difference in the equivalent applied moment *(ΔM)*.

$$\Delta M = \Delta\sigma\left[ntb_c\right](h_{Al} + \frac{nt}{2}) \quad . \tag{12}$$

(2) Composite plate buckling

In our model the plate can be assumed as simply supported around the edges, and subjected to one edge load. It is assumed that the number of half-waves of the buckled shape along the loading direction (*m*) is equal to 2 and the number of waves along transversal direction (*n*) is equal to 1. According to these conditions Equation (13) can be applied for the model.

$$\left(\frac{b_c}{nt}\right) \le \sqrt{\frac{\pi^2}{6\sigma_{\max}\left(1-\nu_{xy}^b\nu_{yx}^b\right)}\left[\sqrt{E_x^bE_y^b} + E_x^b\nu_{xy}^b + 2G_{xy}^b\left(1-\nu_{xy}^b\nu_{yx}^b\right)\right]} \quad , \tag{13}$$

where: $\sigma_{\max}$ the maximal stress in the *CFRP* lamina, b_c the width of the *CFRP* plates, *t* the thickness of one layer, *n* the number of layers of the *CFRP* lamina $E_x^b, E_y^b, G_{xy}^b, \nu_{xy}^b, \nu_{yx}^b$ are the laminate bending moduli.

(3) Web buckling in the Al profiles

$$\frac{h_{Al}}{t_w} \le 42\sqrt{\frac{235E_{Al}}{240E_{Steel}}} \quad , \tag{14}$$

where: E_{Steel} the Young's modulus of Steel (EuroCode 3 1992) .

(4) Stress in the composite plates

The moment caused by loading of the structure is distributed on the *Al* and *CFRP* components of the structure. It is worth to define this ratio and taking into consideration during the calculation of constraints. The moments on the components can be derived from the inertia-ratio of the components.

$$\frac{E_cM}{E_{Al}n_sI_{Al} + E_cI_c} \cdot \frac{h_{Al} + nt}{2} \le \sigma_{cadm} \quad , \tag{15}$$

where: $\sigma_{cadm} = \frac{\sigma_{tensile}}{\gamma_c}$.

The admissible stress can be determined from the tensile strength of a composite layer ($\sigma_{tensile}$) with a safety factor (γ_c =2).
The distributed load can cause a transversal bending but it is neglected during the calculation.

(5) Stress in the Al profiles

The admissible stress can be determined from the tensile strength of a aluminium profiles (f_y =240 MPa) with a safety factor (γ_{Al} =2).

$$\frac{E_{Al}M}{E_{Al}n_sI_{Al} + E_cI_c} \cdot \frac{h_{Al}}{2} \le \sigma_{Aladm} \quad , \tag{16}$$

where: $\sigma_{Aladm} = \frac{f_y}{\gamma_{Al}}$.

(6) Size constraints for the design variables (physical limits)

Size constraints for the design variables [number of layers in the laminate (*n*), number of stiffeners (n_s), cross section of stiffeners (h_{Al}, t_w)] are the followings:

- Deck plates: Number of layers in the laminate at least 2 pieces (n_{lower}), maximum 32 pieces (n_{upper}) due to manufacturing and economical reasons:

$$n_{lower} \le n \le n_{upper} \; .$$

- Number of stiffeners: the structure is stiffened only in longitudinal direction. In case of simple cellular structure (Fig. 1.) there are 2 stiffeners.

  Number of stiffeners is at least 2 pieces (n_{Slower}), maximum 10 pieces (n_{Supper}) in case of multi-cellular structure due to manufacturing reasons:

$$n_{Slower} \le n_S \le n_{Supper} .$$

- Dimension of stiffeners: different SHS profiles were selected from cross sections available in catalogues.

  From point of practical view the following limits were suggested for the cross sections dimensions:

$$h_{Allower} \le h_{Al} \le h_{Alupper} ,$$

where $h_{Allower}$=10 mm, $h_{Alupper}$=100 mm. For these heights the following values are consistent for wall thicknesses (t_w):

$$t_{wlower} \le t_w \le t_{wupper} ,$$

where t_{wlower}=2 mm, t_{wupper}=6 mm .

2.3.4. Mass- and cost optimization was completed for simple- and multi-cellular sandwich structure for case of different layer sequences and layer numbers by Rosenbrock-Hillclimb nonlinear optimization algorithm. The parameters to be optimized in case of multicellular sandwich structure were the number of stiffeners, number of layers and geometrical parameters of *Al* profiles.

Optimization of a given structure (Figure 2.) was completed based on the defined objective functions. The mass and cost of the optimal sandwich structure were compared to an optimized total steel cellular structure with a same area and loading conditions [optimization of steel cellular plate was published by Jármai, Farkas, Petershagen].

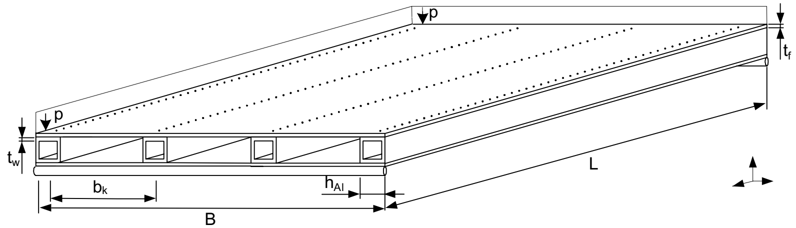

Figure 2. *Multicellular sandwich plate*

The cost of the optimized sandwich structure is higher by 89 % compared to the the cost of total steel structure, but the mass of the optimized sandwich structure is lower by 87 %.

The conclusion of this comparison is that the application of composite components and structures is suggested in those industrial applications where the mass saving is the main design aim and the cost is only secondary.

* Jármai K., Farkas J., Petershagen H.: Optimum design of welded cellular plates for ship deck panels, Welding in the World, Vol. 43., 1999, pp. 50-54.

3. NEW SCIENTIFIC RESULTS (THESES)

1. Elaboration of a calculation method for deflection of simply supported pultruded FRP (FRP = fiber reinforced plastic) I- and box beams. Demonstration of available weight reduction by the application of pultruded FRP beams compared to steel I- and box beams.

2. Results obtained from examination of a cellular plate constructed from FRP deck plates riveted to the upper and lower flanges of two aluminium (Al) square hollow section (SHS) profiles:

2.1 Elaboration of methods for calculation of static stress and deflection and verification of these by measurements.

2.2 Elaboration of finite element calculations for verification of static deflection calculation of the analysed sandwich structure.

2.3 Determination of vibration damping of analysed structure by the Oberst measuring method to show the vibration damping effect of FRP deck plates.

2.4 Construction of cost objective function of sandwich structure by the examination of material- and manufacturing cost components.

2.5 Determination of optimal parameters resulting minimal weight and minimal cost of the analysed sandwich structure by Rosenbrock Hillclimb nonlinear optimization algorithm taking the defined design constraints into consideration.

3. Results obtained from examination of a multi-cellular plate constructed from FRP deck plates riveted to the upper and lower flanges of more aluminium square hollow section (SHS) profiles:

3.1 Construction of cost objective function of a multi-cellular sandwich structure by the examination of material- and manufacturing cost components.

3.2 Determination of optimal parameters resulting minimal weight and minimal cost of the analysed multi-cellular sandwich structure by Rosenbrock Hillclimb nonlinear optimization algorithm taking the defined design constraints into consideration.

3.3 Comparison of weight- and cost optimized composite multi-cellular sandwich structure (FRP deck plates riveted Al square hollow section profiles) to full steel multi-cellular structure which has the same parameters. Result of the comparison is that the optimal FRP sandwich structure can be constructed with 89% extra cost, but the weight of it is less with 87% compared to the total steel structure.

4. PUBLICATIONS ON THE SUBJET OF THE THESES

Publications in Hungarian:

- *Szálerősítéses műanyag kompozit szerkezetek vizsgálata,* Diplomamunka, 1998, 72 oldal
- *Szálerősítéses kompozit szerkezetek statikus és dinamikus vizsgálata*, Doktoranduszok Fóruma, Miskolc, 2001. november 6., pp.: 85-90, ISBN 963 661 482 2
- *Szálerősítéses műanyag I- és szekrény-szelvényű tartók optimális méretezése*, ME TDK dolgozat, Miskolc, 1999., 27 oldal
- *Szálerősítéses műanyag I- és szekrény-szelvényű tartók optimális méretezése*, XXIV. Országos TDK konferencia, TDK dolgozat, Budapest, 1999., 27 oldal

Publications in English:

- *Analysis of a Composite Structure*, MicroCAD 2000. konferencia kiadvány, Miskolc, 2000., 81-84 oldal
- *Analysis and optimum design of fiber reinforced composite structures*, (társszerzők: Groenwold, A. A., Jármai, K., Farkas, J.) World Congress on Structural and Multidisciplinary Optimization, 2001. jún. 4-8. Dalian, China. Proceedings, pp. 381-382
- *Analysis and optimum design of fiber reinforced composite structures*, (társszerzők: Groenwold, A. A., Jármai, K., Farkas, J.) World Congress on Structural and Multidisciplinary Optimization, 2001. jún. 4-8. Dalian, China, CD Edition 6 page, Liaoning Electronic Press, 2002, Proceedings on CD file:///E:/data/papers/151.pdf, 5 p. ISBN 7-900312069-2

- Static and dynamic analysis of fiber reinforced composite structures, (társszerzők: Jármai, K., Farkas, J.) 3rd International Conference of PhD Students, Miskolc 13-19 Aug. 2001., pp. 253-258
- *Optimal design of a composite multicellular plate structure,* Journal of Computational and Applied Mechanics, A Publication of the University of Miskolc, Miskolc University Press, 2004, Vol. 5., No 1., pp. 79-88
- *Analysis and optimum design of fiber reinforced composite structures*, (társszerzők: Groenwold, A. A., Jármai, K., Farkas, J.) Journal of Structural and Multidisciplinary Optimization, 2004, No. 28, pp. 170-179