University of Miskolc

Investigation of Coupled Thermo-Mechanical Wearing Problem with *hp*-version of the Finite Element Method

Ph.D. Thesis summary

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Summary

A coupled thermo-mechanical contact problem has been investigated taking into account the effects of the generated heat and the geometrical modification due to friction-induced wear. The mechanical-, contact-, and heat conduction problems have been first solved separately and then in a coupled fashion.

An early analytical solution of the mechanical contact problem is credited to Hertz (1882), evoking the presumption that the size of the contact region is significantly smaller compared to the bodies in contact.

Numerical investigations of the mechanical contact problem were initiated in the 1930s, with the first variational principle published by Signorini in 1959. A cornerstone of Signorini's idea was that the contacting bodies, when they are pressed together, can separate form each other. The theory was further improved by Fichera, who proved the existence and uniqueness of the solution of the elastic contact problem without friction. Numerical calculations employing the finite element method (FEM) have first been applied for solving contact problems in the 1970s. These computations were mostly based on the so called h-version of the method, where the unknown fields are approximated using first-, or second-order polinomials and the number of elements are increased to attain higher accuracy. Another approach is to increase the degree of the approximation polinomials while keeping the number of elements constant leading to p-version FEM. In numerical methods, when higher-order polinomials are required, the Legendre polinomials are frequently the functions of choice due to their advantageous properties. An other unique characteristic of these methods, is that the geometry of the bodies under consideration is approximated by the same functions as the unknown fields. With the help of higher-order approximation functions an accurate numerical resolution of the contact problem can be obtained.

From the numerical point of view, a particular difficulty arising in contact problems is the discontinuity of the contact pressure derivatives along the boundary of the contact region. Since the pressure depends on the displacement field, which is approximated by higher-order polinomials the numerical solution can exhibit unphysical behavior in the vicinity of contact discontinuities, thus special treatment is required.

Gabbert suggested reverting to linear shapefunctions at discontinuities while keeping the higher-order approximation at smooth regions. An other technique due to Volpert is to apply singular functions, while Paczelt employed a node positionig technique without the need to employ special shape functions. In his work, the positioning process consists of two stages, a rough and a fine positioning and the desired accuracy is controlled by error indi-

cators. However, during node positioning, the finite elements can become distorted resulting the degradation of accuracy. In the present work, a geometrical method has been developed, employing p-version FEM and mesh grading near the contact discontinuity. To avoid the distortion of the elements remeshing has also been employed.

If contacting bodies are sliding on each other, the bodies experience wearing and heat is generated. The wear of the bodies can be described with the modified Archard's equation.

Due to wearing the finite element grid is remeshed to avoid the vanishing of the small elements on the contact surface after a short time. Due to the friction and the heat generated on the contact area, the bodies warm up resulting in the expansion of the contacting bodies, which is in turn influences the solution of the mechanical contact problem. Because of the wearing process the finite element meshes are regenerated in consecutive time steps, but during the heat conduction step the meshes remain the same. This gives rise to the problem of interpolation of the temperature field between different meshes. Several interpolation techniques have been suggested in the literature, however, employing only h-version FEM. In the present work such an algorithm has been developed for p-version.

Since the above thermo-mechanical contact problem exhibits a strong coupling between the temperature and displacement fields, an iterative predictor/corrector procedure has been followed.

The contributions of the present work can be summarized as follows:

1. A numerical procedure for the solution of the mechanical contact problem employing the p-version FEM has been developed applying a new geometrical method for node positioning.

2. The application of graded elements at the problematic contact boundary has been proven to be successful in significantly reducing spurious oscillations compared with non-graded meshes.

3. During solution the finite element mesh is regenerated to account for wearing effects. To further reduce the inaccuracy due to oscillations near the contact boundary, the method of least squares has been applied to determine the new shape of the contacting surfaces.

4. An iterative algorithm has been developed to solve the wearing problem. The size of the timesteps were determined by numerical experiments.

5. A method based on least squares has been developed for transferring the temperature field between different finite element meshes.

6. An iterative algorithm has been developed to numerically solve a coupled thermomechanical problem applying operator splitting and all of the above techniques. An advantage of the algorithm is the ability to enforce the convergence of the displacement and temperature fields.

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