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FACULTY OF MECHANICAL ENGINEERING AND INFORMATICS

MATHEMATICAL MODELLING OF TOOTH SURFACES OF CURVED TOOTH BEVEL GEARS AND APPLICATION POSSIBILITIES OF MODELS

Booklet of PhD Theses

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1. INTRODUCTION

Nowadays, an indispensable tool in gear design is the mathematical modeling of tooth surfaces. This is especially true for drive pairs with complex geometry, such as bevel gears with curved teeth. In the industry, more and more attention are being paid to unwanted vibrations and the possibilities of eliminating them. Two significant areas can be mentioned, which are the focus of research.

These two areas are the testing of noise pollution caused by surface machining and connection defects, as well as service life and failure. During the design and construction phase, harmful vibrations in the system can be reduced by ensuring more precise connection conditions, which on the one hand ensures quieter operation of the equipment and, on the other hand, reduces wear and tear on components due to better contact conditions.

Tooth surface models can be used in many areas of design: detection of geometric anomalies, control of manufacturing accuracy, operational simulations, examination of connection relations, load capacity tests, etc.

Let us mention a few typical areas without claiming completeness, where accurate gear tooth design is of great importance.

Engine design for helicopters and aircraft (critical area for safety and service life).

For land vehicles (noise reduction of powertrain components, increasing efficiency and increasing service life).

For watercraft (for military and luxury vehicles, quiet operation and service life are the main considerations, while for civilian applications, such as passenger and cargo ships, high load capacity and service life are the primary considerations).

2. INTRODUCTION

The aim of the thesis is to use the mathematical modelling of tooth surfaces as a tool to propose improvements to the following subfields of curved tooth bevel wheel design:

- design of low-noise bevel gears with a pair of drive theoretically free from kinematic defects,
- determination of surface points as reference values for measurement on a 3D Coordinate Measuring Machine,
- production of tooth surfaces as point clouds that can be imported into CAD software, as the basis for the CAD model for finite element analysis,
- accurate calculation of head band thickness for examination of tooth acuteness, replacement of an approximate method,
- accurate calculation of contact number, in order to replace the approximate methods.

The figures, the figure numbering and reference numbering in the booklet are the same as in the thesis.

3. SHORT DESCRIPTION OF THE RESEARCH WORK AND ITS RESULTS

1. THESIS
Calculation of the headband thickness of curved tooth bevel gears

One of the typical forms of damage is tooth head fragmentation, which is caused by tooth sharpening. This phenomenon can be avoided by creating an appropriate head thickness. The starting point for the research was AGMA 929-A06 recommendation and the ANSI/AGMA 2005-D03 design standard, which provide guidance on calculating the headband thickness of bevel gears. The essence of the method is to replace bevel gears with a process known as the Tredgold approximation, the so-called virtual cylindrical gears. I applied the two calculation methods presented in Chapter 8 of my dissertation to the gear in the number example.
See the detailed calculations in Annexes 4 and 5.

10.4. table*. Results obtained by the AGMA method.*

Designation		Sign	Data
Head band thickness in the normal plane, (mm)	**outer tooth end**	s_{vane}	**2.765**
	middle of the tooth	s_{vanm}	**2.96**
	inner tooth end	s_{vani}	**2.503**

10.5. table*. Results obtained from surface models.*

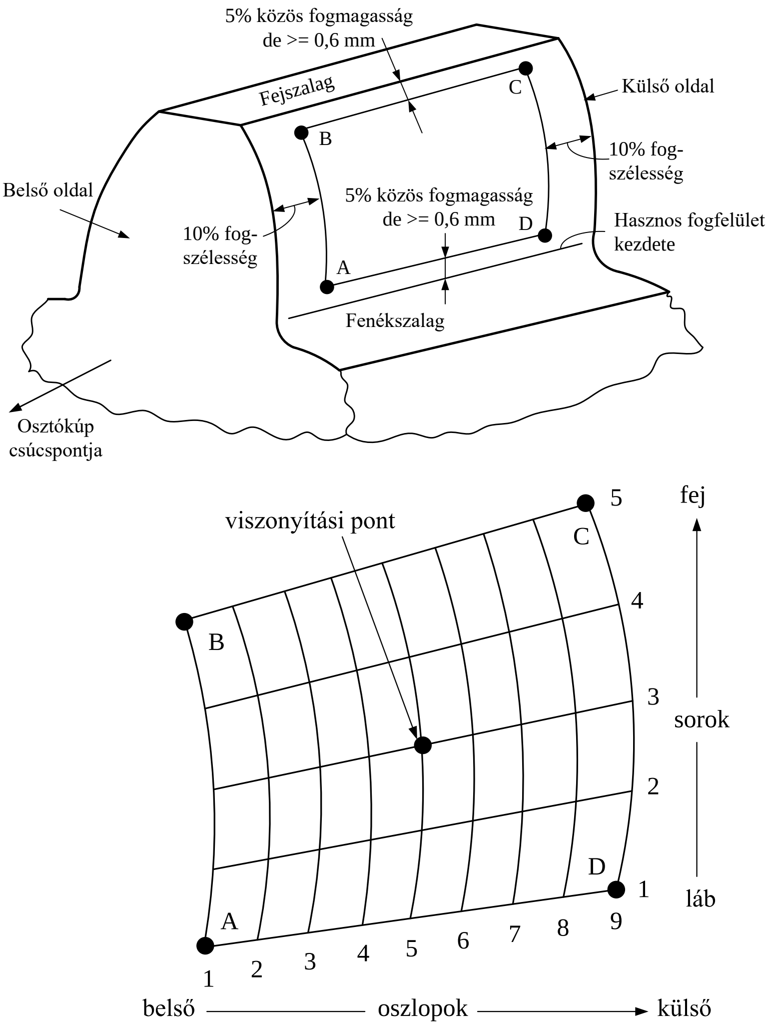
Designation		Sign	Data
Head band thickness in the normal plane, (mm)	**outer tooth end**	s_{vane}	**2.894**
	middle of the tooth	s_{vanm}	**3.002**
	inner tooth end	s_{vani}	**2.599**

Conclusion: the above results clearly show that the approximate method recommended by AGMA gives a lower value at all three test sites than the accurate procedure based on tooth surface models. The smallest difference occurs in the middle of the tooth, 1.4%, and at the two ends of the teeth 4.4% and 3.7%. Of course, no general conclusion can be drawn from a single test on a single gear, but it is recommended to use the method that offers higher accuracy for more demanding calculations.

2. THESIS

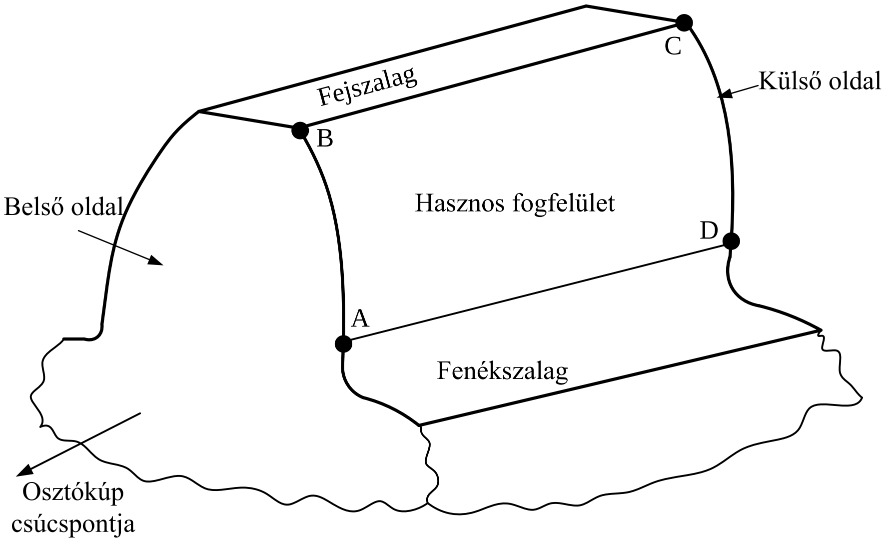
Determination of surface points for finite element testing and measurement on a CMM

VEM simulations are based on the CAD model of the gear, which requires knowledge of theoretically accurate tooth surfaces. The ANSI/AGMA 2009-B01 standard proposes the solution shown in Figure 7.1 for the placement of a point mesh for CMM measurement of bevel gears. This method serves as the basis for comparison.



7.1.-7.2. figure. Point Mesh Placement [ANS01] and Grid Point Allocation

During further examinations, the dot mesh was extended to the useful tooth surface, as shown in the following Figure 7.3.

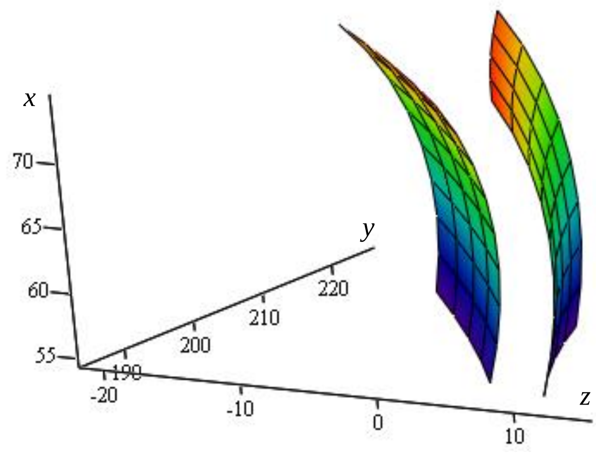


7.3. figure. Place a point mesh on the extended surface

Based on the mathematically described surface models described from Chapter 4 onwards, the method described in Chapter 7 was applied to the large wheel of the sample gear. The dot mesh thus determined allows further examination of the entire tooth surface. Detailed calculations are given in Annex 6 and calculated mesh points are summarized in Tables 10.6 and 10.7.

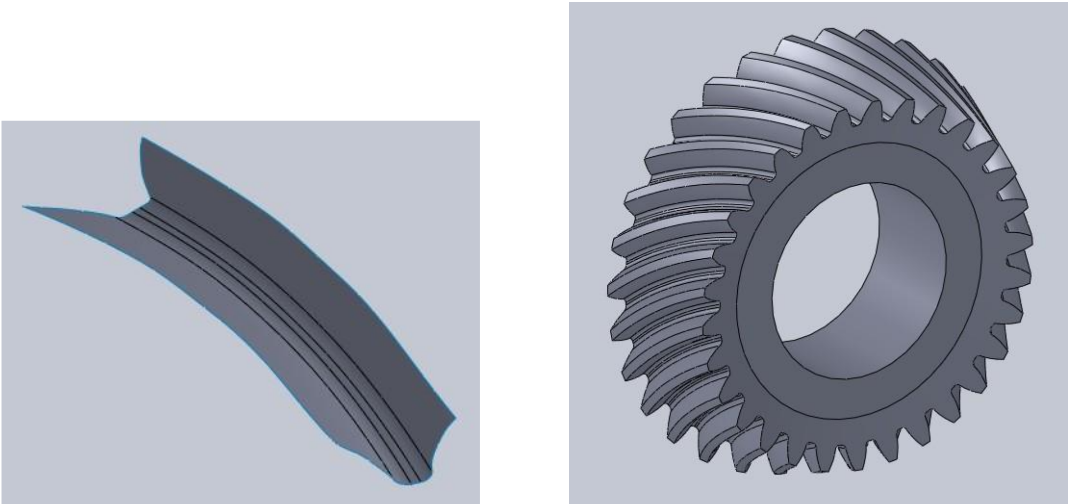
3. THESIS
Creating a CAD model for VEM examination based on the knowledge of tooth surface points

Using the calculations made so far with the help of tooth surface models, the coordinates of the points on the tooth surface and the coordinates of the normals at the given points can be generated with the help of the mesh point coordinates described in thesis 2. Detailed calculations and data are given in Annexes 7 and 8. The results are summarized in Tables 10.8 to 10.17. Using the data of tables 10.7-10.11, I created a surface model showing the two sides of teeth in real position (Figure 10.1.).



10.1. figure. Surface model of the two tooth sides

Processing the data with CAD software, I created the surface model of the tooth trench for finite element analysis (Figure 10.2.), then the body model of the gear (Figure 10.3).



10.2.-10.3. figure. Teeth trench surface model and bevel gear body model

4. THESIS
Computerized procedure for contact analysis

For the examination of curved tooth bevel gear pairs, I created a computer program analyzing tooth contact, the purpose of which is to determine the coupling line on the tooth surface. The program also gives you the option to specify the contact number.
Program input:
- tool and machine setting data for both gears,
- mathematical models of tooth surfaces using the method described earlier
- to determine the contact points, the angle of rotation ϕ_1 of the small wheel is treated as an input parameter and varied in the interface range of a pair of teeth

To plot the switch line, it is advisable to switch to cylindrical coordinates, since radial R and axial L coordinates give a more visual solution in plane than using Cartesian coordinates.

$$R_i(\varphi) = \sqrt{x_i^2(\varphi) + y_i^2(\varphi)}, \quad L_i(\varphi) = z_i(\varphi), \qquad i = 1,2 \text{ and } \varphi = \phi. \qquad (9.17)$$

I used the data provided by the software to perform the calculations found in my dissertation and attachments.

5. THESIS
Determination of the real contact number

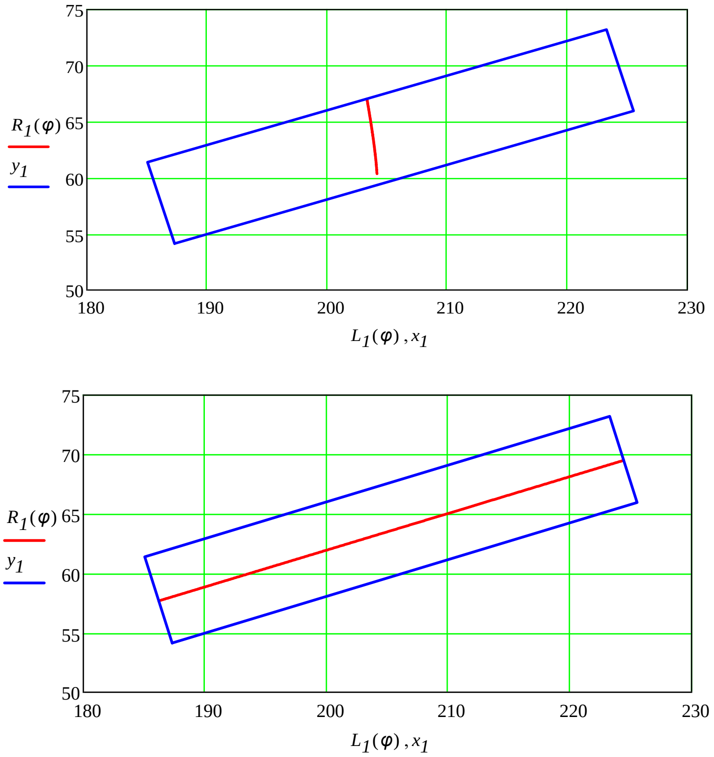
The most common calculation methods in technical practice and literature are:
- calculation of overlap on the dividing plane of the imaginary plane wheel
- overlap calculation based on virtual cylindrical wheel
- overlap calculation according to AGMA

My proposed new solution:
- own procedure considering localized contact pattern

With the data of the sample gear, the contact number was determined according to the traditional methods accepted in the literature (Annex 9) and with the solution based on the switch line. The latter was applied to the two production models presented in Chapter 5 (detailed calculations for production model 1 in Annex 10 and for model 2 in Annex 11). The results of the calculations are summarized in Table 10.18.

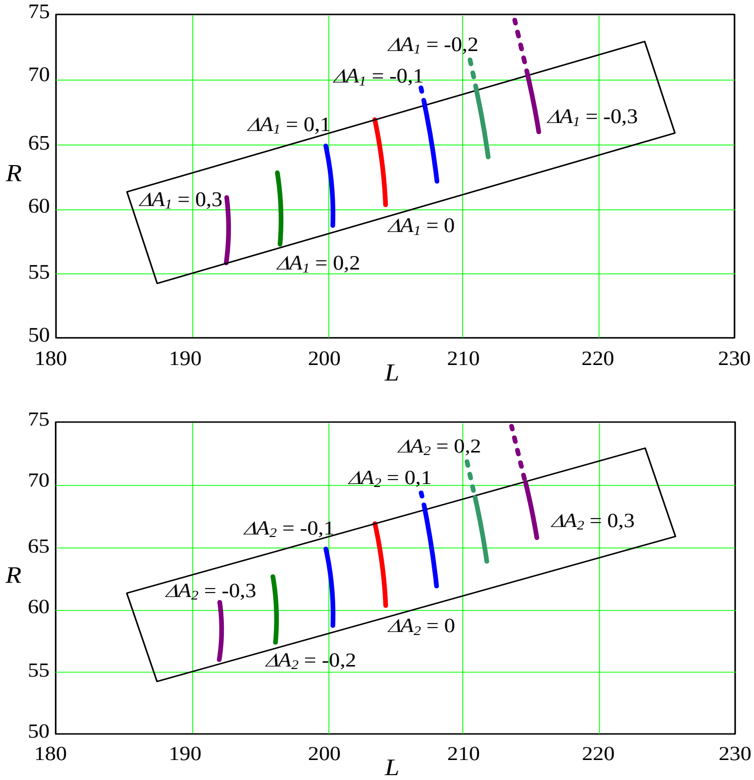
10.18. table. *Contact numbers*

Calculation method	Profile Contact number	Overlap	Full contact numbers
Traditional	1.362	1.684 1.677 1.685	**2.166** **2.160** **2.166**
From contact line for model 1.	1.326	-	**1.326**
From contact line for model 2.	-	1.682	**1.682**

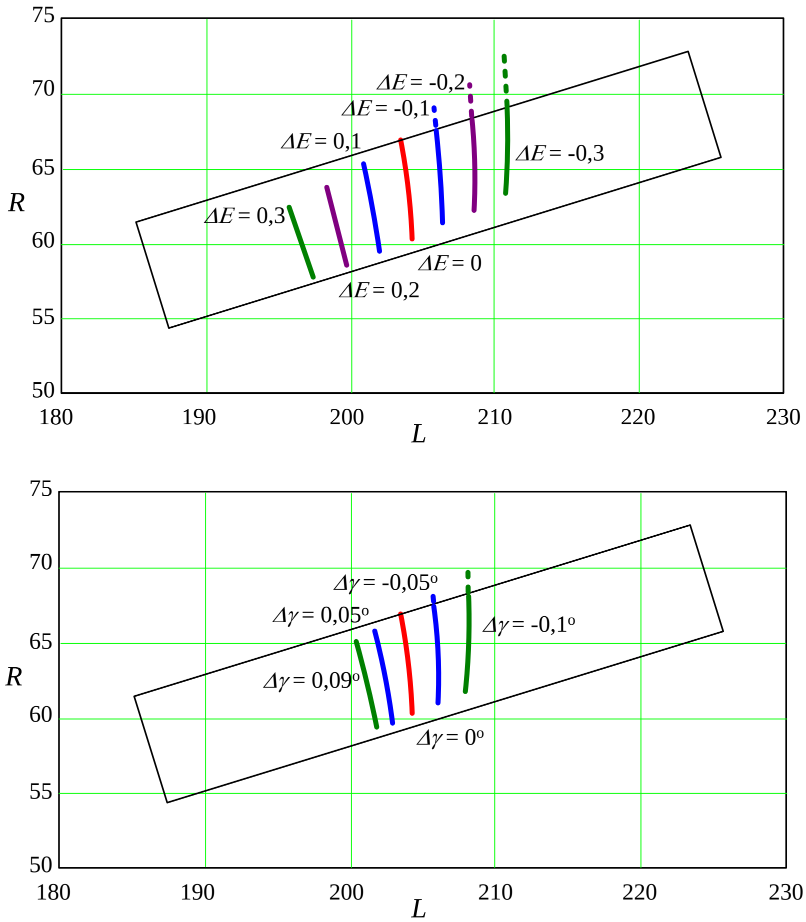


10.4.-10.5. figure. Contact line on the pinion for production models 1 and 2

Changes of contact line due to assembly errors



10.8.-10.9. figure. Change of contact line in case of axial mounting failure of the pinion and the gear



10.10.-10.11. figure. Change of contact line in case of shaft distance and shaft angle failure

4. CONCLUSION

After presenting the above procedures and results, with calculations based on accurate geometric models we can create more accurate connection values, 3D and simulation models. By applying these procedures, we can examine the operating conditions of curved tooth bevel gears in more detail, thus we can develop their operating parameters.

5. NEW SCIENTIFIC RESULTS – THESES

T1. Based on the mathematical modeling of tooth surfaces, I developed a new method for calculating the headband thickness of curved tooth bevel gears. The previously known procedure was based on the replacement cylindrical gear pair method, that was an approximation method. The new method also provides an accurate solution for calculating the thickness of the head band by accurately describing tooth surfaces, enabling the detection and elimination of tooth sharpening during the design phase.

T2. Using the mathematical modeling of tooth surfaces, I created the point mesh necessary for checking the tooth surfaces on a coordinate measuring machine, which I inserted onto the theoretical tooth surface and determined the coordinates of the given points, which serve as reference values. By comparing the theoretical tooth surface data with the measured values of the finished gear, the accuracy evaluation can be performed.

T3. As a result of the procedure presented in thesis 2, the coordinates of the tooth surface points are available on both sides, which can be used to produce a CAD model of the gear suitable for finite element analysis by surface modeling in a suitably chosen CAD system.

T4. For the analysis of tooth contact, I developed a computerized procedure, which is suitable for determining the location of the connection points, to produce the contact line, and for examining the effect of adjustment errors on the connection.

T5. With the computer procedure presented in thesis 4, I developed a new method for determining the contact number of curved tooth bevel gears. By modeling real-world operation, the contact number can be determined precisely. The calculation methods available in the literature so far are based on the replacement cylindrical gear method and are not suitable for considering the localized carrying image.

6. LIST OF PUBLICATIONS RELATED TO THE TOPIC OF THE RESEARCH FIELD

(1) Várkuli Miklós Gábor; Bognár Gabriella; Szente József

New Top Land Computing Method for Spiral Bevel Gears

PERIODICA POLYTECHNICA-MECHANICAL ENGINEERING 67 : 3 pp. 1-7. , 7 p. (2023)

(2) Várkuli Miklós Gábor; Bognár Gabriella; Szente József

Determinaton of tooth surface points on bevel gears for checking on a coordinate measuring machine

DESIGN OF MACHINES AND STRUCTURES 13 : 1 pp. 131-139. , 9 p. (2023)

(3) Várkuli Miklós Gábor; Bognár Gabriella; Szente József

Contact ratio of spiral bevel gears

LECTURE NOTES IN MECHANICAL ENGINEERING Vehicle and Automotive Engineering 4 pp. 103-110., 8 p. (2022) (Q3)

(4) Várkuli Miklós Gábor; Bognár Gabriella; Szente József

Kúpkerék fogfelületek matematikai modellezése végeselemes vizsgálathoz

Mathematical model of spiral bevel gears for finite element analysis

GÉP 2022/3-4 pp. 98-103., 6 p. (2022)

(5) Várkuli Miklós Gábor; Bognár Gabriella

History of Gleason Works spiral bevel gear technology

DESIGN OF MACHINES AND STRUCTURES 12: 2 pp. 146-152., 7 p. (2022)

(6) Várkuli Miklós Gábor

Development of a gear drive designer software

DESIGN OF MACHINES AND STRUCTURES 9: 2 pp. 56-60., 5 p. (2019)

(7) Várkuli Miklós Gábor

Improved accuracy gear tooth CAD modelling

DESIGN OF MACHINES AND STRUCTURES 9: 2 pp. 51-56., 6 p. (2019)

(8) Várkuli Miklós Gábor

Fogazattervező program funkcióinak bemutatása és további fejlesztési lehetőségei

MULTIDISZCIPLINÁRIS TUDOMÁNYOK: A MISKOLCI EGYETEM KÖZLEMÉNYE 10: 1 pp. 94-98., 5 p. (2020)

(9) Várkuli Miklós Gábor

Evolvens fogprofil pontosságának javítása 3D CAD modellen

MULTIDISZCIPLINÁRIS TUDOMÁNYOK: A MISKOLCI EGYETEM KÖZLEMÉNYE 9: 2 pp. 96-100., 5 p. (2019)

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