UNIVERSITY OF MISKOLC
FACULTY OF MECHANICAL ENGINEERING AND INFORMATICS

COMPUTER BASED CONSTRUCTIVE GEOMETRIC AND ANALYTICAL DEVELOPMENT OF THE MANUFACTURING GEOMETRY OF WORM GEAR DRIVE PAIRS

PhD DISSERTATION THESES

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1. INTRODUCTION

Developments in production technology, quality assurance, and production geometry at the University of Miskolc for more than half a century are the defining area of worm drive research. Among the scientifically outstanding results related to my research, I should mention **István Drahos** [19-21], **József Drobni** [22-23], **Dudás Illés** [24-27], **László Dudás** [28-29], **Imre Lévai** [40-41], **István Sipos**[61-62], the work of **József Tajnafői** [70] and their students. I carried out my research work in the field of worm gear drive research at the Worm Gear Science School founded by Professor Illés Dudás, under academic supervision of **Zsuzsanna Óváriné Balajti**. As an IT member of the research group, I developed the new procedures described here using the possibilities of the simulation mathematical analysis toolbox in the field of manufacturing geometry development. My research work, summarized in the dissertation, was prepared to promote engineering innovation activity by expanding the IT-enhanced mathematical toolbox. Founded in the Institute of Manufacturing Sciences at the University of Miskolc, the research group dealing with worm gear pairs, which also operates scientifically as a school, has so far produced 11 doctoral dissertations, hundreds of scientific publications, quite a few patents, and participated in numerous OTKA researches [80-85].

The theses presented in the dissertation were the result of several years of research with the support of my supervisor **Zsuzsanna Óváriné Balajti**. The regularly published research results were presented and discussed at scientific conferences.

The requirement for industry is to comply with the prescribed production accuracy during the production of the worm surfaces and the related gear, which must be ensured by the promotion of science and the development of technology [3, 16, 25, 27, 30-32, 56, 60, 72-74 , 81]. Wear and resharpening of tools, as well as machine settings with limited accuracy, can cause distortion and shape errors of the manufactured tooth surfaces. This tool, in the case of serial production of the gear connected to a cylindrical and conical worm, is the hob, which has an extremely complicated geometry and is an extremely expensive tool. At creating the cutter, it is advisable to enable it to be resharpened as many times as possible. Its design not only requires extraordinary knowledge of production geometry, but also requires serious mechanical, technological and, last but not least, geometric and mathematical knowledge [2, 12, 14, 16, 24-27, 30, 32, 34, 35, 47, 53 , 56, 60, 81].

1.1. The subject of the research

Based on the researched literature and the results of my own research work in this area, the topic of the dissertation was determined, which covers manufacturing geometry analyses of the relationship between the tool edge geometry and the manufactured surface in the key area of tool geometry, including:

1. **Constructive geometric and analytical further development of the mathematical model** developed for the production geometry analysis of conical and cylindrical worm gears **built into software**.
2. In order to set the **cutting edge of the worm gear hob** to a **reference position** and perform a **wear test**, in order **to ensure the reconstruction from its two images**, the exact mathematical determination of the positioning of the CCD cameras **based on the theoretical analysis of the Monge representation**.
3. **Examination of the spatial distortion of the cutting edge curve** of the worm gear hob during machining.
4. **Mathematical determination** of the **adjustment parameters** of a re-sharpened hob.

5. Analysis of the effect of **changed geometrical conditions** due to the re-sharpening of the worm gear hob **on the tooth surface of the worm gear** based **on the principle of enveloping.**

1.2. Applied tools

In connection with my simulation constructive geometry development, I used the tools of complex geometry, linear algebra, differential geometry, representational geometry, computer geometry and production geometry. I wrote the computer programs in the C environment.

1.3. The area of the discussed researchfield and determination of the main objectives

The description of research topic:

Today, it is impossible to practice advanced engineering sciences without a comprehensive tools of computer-supported mathematics. These modern mathematical procedures support the solution of today's engineers' rather diverse and complex technical challenges, and at the same time promote the process of both their research and development innovation activities.

The subject of my research work is the computer-supported mathematical, geometric, and production geometry development of conical and cylindrical worm drives, of which I focus on the development related to the subject area of descriptive geometry.

Mathematical developments are defined by the fact, that those have to describe the technological process under investigation, and those are developed in parallel with the direction of the development of technology, in accordance with the requirements, that it should be meaningful, useful, and systematization.

In the dissertation, I also discuss one of the relevant elements of tool geometry, the analysis of the relationship between the edge geometry and the quality of the manufactured surface.

The background of my research was provided by DIFI-CAD Engineering Agency, Research, Development Trade and Service Ltd., which has a cooperation agreement with the University of Miskolc.

The determined objectives:

1. *The problem that was occurred in the research work:*

The simultaneous geometric and analytical determination of the contact curves of the conical and cylindrical worm surfaces and their related surface pairs arose during the solution of both design and construction tasks. When machining worm surfaces with a grinding wheel, it causes a problem that the diameter change due to wear of the grinding wheel, leads to a reduction in centre distance, which requires a different wheel profile to create the same mathematically defined worm surface. During the machining of the conical worm surface, the change in the pitch angle of the conical worm fitted to the dividing cone entails a change in the axis angle of the worm axis and the machining grinding wheel.

Objective:

Further development of the mathematical model of worm gears, simulation constructive geometric elaboration to determine the contact curve with the connected gear and the changing contact curve due to wear during machining with the grinding wheel.

2. *The further problem was occurred in our research work:*

The fundamental problem of traditional inspection methods of wear is that the surfaces are evaluated by their two-dimensional sections and their plane projections, so the effects of the plane projections of dimensional errors in different directions are only taken into account as a cumulative effect.

Objective:

In order to check the wear of the cutting edge of the worm gear hob that works on the tooth surface of the worm gear, it is necessary to set it to the same reference position during production. During the CCD cameras positioning for creating the Monge mappings taken from two mutually perpendicular directions must be ensured the reconstructability of the intersection curve between the face surface of the hob tooth and the foot cylinder, as well as the cutting edge curve. The criterion of the reconstructability of the curves must be ensured by choosing the projection directions from the bijective parts of the Monge cuboid, respected both spatial curves.

3. *The further problem was occurred in our research work:*

The fundamental element of tool geometry is the relationship between the size and shape of the cutting edge during wear, the analysis of its effect on the machined tooth surface can be a cost-effective solution during production within the given tolerance.

Objective:

Elaboration of a procedure for measuring the curve of the hob cutting edge with two CCD cameras based on the principles of constructive descriptive geometry.

4. *The problem that was occurred in the research work:*

Determining the relationship between the geometric parameters of the hob re-sharpened due to wear based on the characteristics of the connection of the worm gear drive.

Objective:

Analysis of the effect of the half-opening angle of the conical drive pin on the pitch fluctuation when machining the conical worm surface with axial adjustment. Mathematically exact determination of the profile of a geometrically correct drive pin in the form of an explicit equation instead of a series of points to eliminate the thread flutation in the manufacture of a conical worm surface with aadjusted shaft.

5. *The further problem has occured in our research work:*

During the machining of the worm gear with a hob, the change in the centre distance due to the re-sharpening of the hob, and the change in the movement conditions due to the change in the thread pitch angle on the worm causes profile distortion on the gear, so the need to measure the profile distortion was expressed during our research work.

Objective:

Analysis of the effect of changed geometric conditions due to the re-sharpening of the hob on the gear tooth surface based on the principle of enveloping. After calculating the tooth surface points of a gear connected to a specific designed and manufactured worm, their spatial location was also presented. By choosing the geometric characteristics of the tooth surface of the worm and the tooth surface of the gear connected to it, it was possible to optimize the lifetime, operation and efficiency of the worm drives.

2. SOME RELEVANT LITERATURE BACKGROUND

2.1. The rise of the gearing theory of spatial driving pairs

Among the publications published around the turn of the century, the writings of **Altmann** [1] and **Distelli** [18] and **Crain**'s work entitled "Schraubenräder mit geradlinigen Eingriffsflächen Werkstattstechnik" and **Stübler**'s "Geometrische probleme bei der Verwendung von Schraubenflächen in der Technik" should be highlighted, in which in which using the tools of descriptive geometry, they reached extremely valuable results in the field of the development of tooth theory. The concept of the vector screw can be read for the first time in **R. Ball**'s book "Theory of Screws" published in 1900. **Distelli** was one of the first to use the general screw motion to determine the tooth surfaces of a gear with two bypassed shafts in his book "Über instantane Schraubengeschwindigkeiten und die Verzahnung der Hyperboloidräder" published in 1904 [18].

Willis [78] stands out in his book "Principles of Mechanism" and **Dudley** [30] in his book "Gear handbook". **Buckingham** [14] and **Wildhaber** [77] gained international recognition in this field with their well-known works. With a kinematic method, the investigations of the principle problems of engagement can be simplified, based on which **Litvin** and prominent representatives of the Russian school of toothing theory such as **Kolchin** and **Krivenko** [39] developed successful procedures for determining engagement and contact criteria, curvature conditions, and interference phenomena. During the design of the screws and their machining tools, the results of gear geometry and connection theory developed by **Gohman, H.I.** [33], and **Litvin, F.L**. [42] and his colleague, **Fuentes, A**. [43], using differential geometry and coordinate geometry procedures for the transformation of coordinate systems are well they can also be algorithmized to solve the production geometry problems of screw surfaces.

Among the Hungarian researchers, **Lajos Szeniczei** achieved further results in this field in his book "Worm Drives" published **in 1957**, as well as **Bercsey, T.** [10-11], **Drahos, I.** [19-21], **Drobni, J.** [22-23] , **Illés Dudás** [24-27], **László Dudás** [28-29] **Lévai, I.** [40-41], **Magyar, J.** [44], **Máté, M.** [32, 47, 71], **Tolvaly Rosca, F.** [35, 46], **Tomori, Z.** [72-74**] Pay, G.** [55] and **Tajnafői, J.** [70]. **Szeniczei** defined the concept of "conjugate surface pair" by proposing the idea of connection, i.e. mutual enveloping [67]. **Magyar, J.** [44] was also the first in the international literature to shed light on the problems of connecting elements with helicoid surfaces. With his integrative approach, **J. Tajnafői** systematized his theory on the technology of teething through the approach of movement mapping parameters [70]. **Drahos, I**. dealt with the theory of the geometry of tools, including the analysis of helicoid surfaces and hypoid bevel gears, and he introduced the concept of production geometry [19]. **Lévai, I.** analysed the multiplicity of spatial drives and, among other things, developed the gearing theory of gear pairs with deviating axis linear surfaces, including the complex issue of designing hypoid drives, and also wrote about manufacturing axoids [40-41]. Using the kinematics method, **Bercsey, T.** dealt with the connection of the globoid worm with a straight tooth surface and the hyperbolic wheel, as well as the investigation of the toroid drives [10-11].

In Germany, **Bilz** in his work "Ein Beitrag zur Entwicklung des Globoidschneckengetriebes zu einem leistungsfähigen" developed the "TU-ME" globoid drive as a kind of version of involute gear worm gear pairs, which can be included in the group of globoid worm gear pairs with cylindrical wheels, for which Drahos I. carried out the theoretical investigation. **Drobni, J.** dealt with the development and research of the grindable globoid screw drive in 1968 in his candidate thesis entitled "Grindable globoid screw drives"

[22]. **Siposs, I.** also achieved excellent results in this area in his candidate's thesis entitled "Globoid drives made without stripping with a worm wheel" completed in 1990 [61], as well as **Pay, G.** in his thesis "Internal worm drives" [55], and **Máté, M.** with extensive research on various driving pairs [32, 47, 71].

Tolvaly Rosca, F. achieved remarkable results in his writings [35, 46] in the field of computer modeling of the contact tooth surfaces of drive pairs. **László Dudás** contributed to the development of the scientific field with the creation of the Reach model and the construction of a completely new grinding machine, with a special option of the kinematic method suitable for the production of the related tooth surfaces. **László Dudás**' Surface Constructor software supports the derivation of the tool surface suitable for the production of the element pairs of different drive pairs, i.e. the related tooth surface, and the design of mechanisms [28-29]. **Z. Tomori** carried out extensive research on the optimal choice of profile offset factors for cylindrical gears with involute teeth [72-74].

I. Dudás developed and patented the screw with a circular arc profile in the shaft section and its manufacturing process, as well as the spiroid drive with a concave profile [24-27, 81-82]. **P. Horák** wrote about his valuable results regarding the favorable range of the instantaneous contact lines in his dissertation entitled "Tribological investigation of worm gear pairs with a circular profile" in 2003 [36]. **I. Dudás** wrote in his book of outstanding value, internationally recognized, about his general kinematics model developed for the manufacturing geometric analysis of the elements of cylindrical and conical worm drives, which was also published in English. In his book "The Theory and Practice of Worm Gear Drives", he wrote in remarkable detail about the connection of worm gear pairs, the analysis of their design, and the production of their components. **Zsuzsa Balajti** revealed functional relationships between the parameters of the tooth profile and the position of the knotes that determine the bearing pattern in the case of a cylindrical worm gear drive with a circular arc profile in the axial section [54]. **Péter Horák** investigated the connection conditions of pairs of worm drives with his computer model [37], and also dealt with the variable ratio of different drives [9-10]. **V. Simon** analyzed the geometrical conditions of both cylindrical and globoid worm drives and their frictional losses, as well as carried out the optimization from the point of view of load capacity on the scheme of the thermotechnical lubrication model [59].

Zsuzsa Balajti further developed Dudás' general mathematical model by making the conical and cylindrical worm with a common axis in order to eliminate the pitch error [45] that occurs during the machining of conical worms in her dissertation entitled "Development of the manufacturing geometry of kinematic drive pairs" [52], and then for the mathematical generalization, the projective relationship has been outlined in the spatial model in the 1st thesis of his habilitation thesis book entitled "Development of connected surface pairs in production geometry, with a representational geometrical application" [54].

2.2. The preludes of the development of constructive descriptive geometry

The mapping created by **G. Monge** (1746-1818) [48] represented a significant advance in the separation of artistic representation and scientifically based technical representation, so that the design and construction of fortifications in the representation was treated as a military secret for more than three decades. In the future, however, anomalies occurred in relation to the mutual clear correspondence with regard to the technical practice in the representation of the space elements by Monge [2, 2-21, 53, 63, 85]. During the representation of the space elements, the expectation of a mutually clear correspondence, i.e., the reproducibility of two ordered perpendicular projections, is a bijective mapping in the language of mathematics. The mapping is bijective if it is injective (that is, the value set elements corresponding to different elements of the interpretation domain are different) and surjective (that is, each element in the value set has an element of the interpretation domain corresponding to the element in the value set). The reason for the anomalies, i.e. ambiguities regarding the mutual clarity, i.e. bijectivity, of the descriptive geometry may be the attitude of Monge, who performed the representation of polyhedra synthetically, whereas he solved the representation of curves as an analytical geometric task. As the analytic geometrical aspects of the Monge projection faded later, the practice switched exclusively to synthetic methods, which could not ensure bijectivity in all cases. The representation of the point is a bijective mapping, that is, the Monge projection maps the space onto the plane mutually unambiguously, but this is not always true for the other basic geometric elements.

The founder of the department in Selmecbánya **J. Hönig**, professor at the University of Technology, discussed the foundations of descriptive geometry synthetically, while the representation of curves and surfaces was discussed analytically [38]. **Géza Petrich** gained great recognition among mathematicians and engineers in his book "Descriptive Geometry" with his synthetic approach [57].

In several articles [20-21], **István Drahos** dealt with the mutual clarity of Monge's representation. In one of them, he writes that "the appearance of computer geometry fundamentally shook descriptive geometry in the scientific sense, making it completely independent of human perception as to whether a descriptive geometry system is mutually clear in terms of mapping and reconstruction. *The reconstruction algorithms sometimes led to contradictions.*"

József Szabó in his article entitled "Eine computergraphische Anwendung des zentralen Einschneide-verfahrens der Darstellen Geometrie (Rekonstrutionsprobleme)" [65] describes the point reconstruction method required for the computer to reconstruct spatial objects from only two images. In his habilitation doctoral dissertation, he develops a point reconstruction method with significantly more calculations, than the previous one from two images of arbitrary positions, which is based on projective invariants, and a sure solution only exists if the two images of the point are known [66].

In technical practice, the task regularly occurs, that components, workpieces, or any other three-dimensional body must be reconstructed from their two two-dimensional images [2, 21, 31, 56, 57]. Due to the peculiarity of the Monge projection, two mutually perpendicular projections are sufficient for the bijectivity of the representation in the case of polyhedra. Geometric bodies represented by points corresponding to each other can also be modeled in different CAD systems [63, 64]. **Mária Nándoriné Tóth** writes in her doctoral dissertation that if the vertices of the polyhedra are not marked, there are also cases where it does not matter which side of the body the third image plane is used for the clear reconstruction of the object [50]. If only two mutually perpendicular projections of the geometric form are available, and the correspondence of the points is not marked on them, then the mapping is very often not bijective [2, 21, 53].

Nowadays, the need to investigate the reconstructability arose in connection with the need to reconstruct the representation in the Monge projection during drawing digitization [20, 53, 63]. In order to ensure the reconstructibility of the two-camera measurement, **Zsuzsa Balajti** developed the correspondence between the camera positions and the points of the Monge brick [53] in her book "Theoretical analysis and application of the Monge representation in engineering practice", in which, among other things, based on the obtained theoretical results, computer programs were prepared, with which Monge projections ensuring bijective representation of helix parts were selected. The programs were also run in some critical cases, and thus the theoretical results became visual. The processing of the calibration of the CCD camera during the OTKA research "Development of CCD camera measurement systems in the field of quality assurance in the mechanical industry" carried out at the University of Miskolc produced serious results [76, 85].

Regarding the correction of distortion of the images taken with the camera, many solutions have been published, among which I mention the thesis "Camera calibration based on photographs" by **István Szűcs** [69].

3. METHOD AND SIGNIFICANCE OF SOLVING THE TASKS

1. Software has been developed for the new, simultaneously constructive geometric and analytical hybrid model, which is suitable for examining the contact curve in case of machining of the worm surfaces with a grinding wheel, and also suitable for analysing of the contact curve of the worm gear drives. As a result of the re-sharpening of the cylindrical and conical worm gear hob, the new hybrid model is also suitable for handling the changing motion conditions due to the change of the axis distance and the pitch angle. The computer geometry mathematical hybrid model was created in the Visual Studio 2019 development environment, based on the characteristics of C++ and OpenGl.

The new hybrid model growed for the development of the manufacturing geometry of conical and cylindrical worms can simultaneously handle geometrical and analytical parameters that change during production, such as the distance and angle between the axes, and also provides the opportunity to determine the contact curves of the gear and grinding wheel connected to the designed worm.

2. For testing, the cutting edge of the hob must be placed into a reference position for each measurement. The intersection curve ***L*** between the front surface of the hob-tooth and the root-cylinder was selected for positioning, as it is not affected by wear. In order to simultaneously ensure that the root-cylinder curve ***L*** and the cutting edge curve ***V*** can be reconstructed from the two images, a complicated trigonometric inequality with three unknowns must be satisfied for each of these two third-order spatial curves. The solutions were determined using a new procedure, according to which they are shown by the surfaces bounding the bijective parts of the Monge cuboid - considering the curves ***L*** and ***V*** - taking into account the discussions during the research work - with the program developed for this purpose. The correct camera positions were selected from the common points of the bijective parts of the Monge cuboid.

The developed procedure simultaneously offers the possibility to solve trigonometric inequalities with three unknowns by looking at two third-order spatial curves, as a result of which the correct camera positions can be determined mathematically with the direction angle parameters chosen from the bijective part of the Monge cuboid.

3. A procedure was developed for the examination of the distortion of a hob cutting edge set to a reference position during machining using the analytically followed representational geometry toolbox. In addition to the spatial location of the cutting edge points, the procedure shows the spatial deviation from the theoretical curve with cameras set in accordance with the re-constructability criteria considering the hob adjusting to the reference position.

All of this can also provide information about the machined surface.

The procedure examines the measurement of wear during machining by determining the spatial location of the cutting edge points, which can be used to directly mathematically model its effect on the machined surface.

4. The hob used for machining the worm gear is a tool whose machining surface is the same as the surface of the worm connected to the gear. The research was developed on the basis of the principle of two-parameter enveloping and the geometrical relationships of the elements

of the worm gear drive pairs in case of machining by the worm gear hob with a circular arc profile in axial section. The re-sharpening after wear of the hob manufactured with special tooth side surfaces results in a reduction of the diameter of the addendum-cylinder and pitch-cylinder, which causes changes in the geometrical conditions at unchanged tooth profile. The re-sharpening of the hob requires the adjustment of the axis and the angle correction between the axes at the same time.

The description of the setting parameters of the re-sharpened hob with a function was developed to examine the manufacturing accuracy of the gear connected to a cylindrical worm with circular arc profile in axial section. By precisely adjusting of the hob, distortion of the gear tooth surface, which is sensitive to geometrical conditions, beyond the tolerance limit can be avoided.

5. Using the new mathematical computer hybrid model, the examination of the contact conditions of the helicoidally surface and the tooth surface of the wheel connected to it - based on the principle of two-parameter enveloping - leads to the examination of profile distortion, which was also presented in the case of a specifically designed and manufactured worm drive.

Using the presented procedure, it is possible to determine the relationship between the profile distortions of the tooth surface of the gear machined under changed geometrical conditions and tool wear with a re-sharpened hob.

The presentation of the mathematical analysis is part of the PhD thesis.

4. SUMMARY

In today's technological environment, the computer-supported or fully automated systems are taking over the design processes of products and production tools. Computer support for modern engineering work is one of the most important conditions for increasing production efficiency and improving product quality. The development of production automation can be promoted by a mathematical computer hybrid model developed for the development of production geometry, which also shows progress in eliminating manufacturing errors. A mathematical computer hybrid model was developed for the production geometry development of conical and cylindrical worms. The model provides an opportunity to design helicoidally surfaces, to determine and analyse the tooth surface of the gear connected to the helicoidally surface. There is also the possibility to examine the contact lines of the helicoid surface and the re-profiled grinding wheel due to wear. The model gives the possibility of the examination of the helicoid surface profile distortion caused by the variable wheelbase and the variable thread pitch angle during the machining of the helicoid surface with the grinding wheel, the purpose of which is to examine the manufacturing accuracy.

During the machining of the worm gear with the hob, a new mathematical procedure has been developed for the positioning of the cameras in order to ensure the re-constructability of the cutting edge curve for the wear measurement with two CCD cameras. At the same time, by placing CCD cameras, it is also necessary to ensure the reconstruction of the front surface of the hob tooth and the cutting curve of the bottom cylinder. To ensure the simultaneous correct adjustment of the CCD cameras to the curve of the hob tooth face surface on the root-cylinder and the cutting edge curve can be reconstructed from only two perpendicular projections, it is based on the points from the bijective common space parts in the Monge cuboid. The direction angles belonging to the point chosen from the bijective regions of the Monge cuboid determine the direction in which the CCD cameras should be positioned. To measure the wear of the cutting edge of the hob during machining, a new procedure has been developed for reconstructing the spatial location of the cutting edge curve from two images taken with CCD cameras using the methods of constructive descriptive geometry.

The cutting edge curve reconstructed from the images has been compared with the mathematically determined curve to compare the spatial deformations with the given tolerance.

A gear connected to a worm with a circular arc profile in axial section is machined with a hob, which is formed from the worm. Due to the wear of the cutting edge of the hob, re-sharpening is necessary. Due to the special design of the hob tooth, the geometry of the cutting edge does not change, but the geometric environment of the motion transfer does. As a result of the re-sharpened hob, the centre distance is reduced and the angle between the axes also changes. The relationship between the geometrical parameters was determined in the form of a function, for the mathematically correct setting.

The worm gear is connected to the worm with a circular arc profile in the axial section. During the machining of the worm gear with the hob created from the worm, correlations regarding the mathematical accuracy of the surface of the gear machined with a tool re-sharpened due to tool wear were revealed in order to determine the correlation between the wear of the tool and the manufacturing accuracy of the gear tooth surface.

5. NEW SCIENTIFIC RESULTS

The new scientific results of the dissertation are summarized in the following theses:

Thesis 1: I created a mathematical computer hybrid model based on the constructive geometric model developed for the production geometry development of the elements of the previous conical and cylindrical worm drive pairs, **which enables the design of helicoid surfaces, the determination of the tooth surface of the connected gear,** and **for the immediate monitoring of changes in geometric parameters**, such as **the distance and angle of the axes**, **during machining with the grinding wheel**, in order to increase production accuracy. [A-1, A-5, A-9, A-14]

Thesis 2.: I developed a new mathematical procedure for the correct positioning of the CCD cameras, regarding the root-curve of the tooth front surface of the **curved hob** on the root-cylinder, **in order to set into a reference position**, and regarding the cutting edge curve **for wear measurement**, to ensure that **it can be reconstructed from only two perpendicular images**, by determining the angle parameters defined **by the bijective parts of the Monge cuboid for both curves**. [Á-3, Á-6, Á-10, Á-13, Á-15]

Thesis 3: I developed a new geometric procedure for measuring the wear of the hob cutting edge for reconstructing the spatial location of the edge curve **from images taken with CCD cameras**, using the methods of constructive descriptive geometry, according to which the cutting edge points reconstructed from the images are compared with the mathematically determined edge **curve in order to compare the spatial deformations with the specified profile tolerance**. [Á-2, Á-7, Á-8, Á-12]

Thesis 4: I explored the relationship between the reduction of the centre distance and the angle of the tool axis in the form of a function **for the adjustment of the re-sharpened hob** with a circular arc profile in axial section, for the mathematically correct adjustment. [Á-7, Á-12, Á-16]

Thesis 5: I determined the relationship between the theoretical tooth surface of the worm gear connected to a worm with a circular arc profile in the axial section **and the tooth surface created during machining with a re-sharpened hob** in order to explore the relationship between tool wear and the manufacturing accuracy of the tooth surface of the gear. [Á-4, Á-9, Á-11,]

6. DEVELOPMENT DIRECTIONS, OPPORTUNITIES

A mathematical computer hybrid model prepared for the production geometry development of cylindrical and conical worms can be further developed to model drive pairs with other geometries.

The positioning of the CCD cameras can also be worked out to ensure the reconstruction of additional space elements at the same time.

Developing a way to compare the distortion of the worm gear with the analytically prescribed surfaces was also formulated as a future goal.

The development of the production geometry of conical screw surfaces in the direction of computer-controlled production is also a challenge requiring multidisciplinary knowledge, which I would like to continue.

7. PUBLICATIONS RELATED IN THE DISSERTATION TOPIC

Journal article published abroad written in foreign language:

[Á-1.] BALAJTI, ZSUZSA ; **ÁBEL, JÓZSEF**: Applying projective geometry in design of worm manufacturing KEY ENGINEERING MATERIALS 581 pp. 77-81. , 5 p. (2014) DOI REAL MIDRA Scopus

[Á-2.] BALAJTI Z. , **ÁBEL J.**: Edge geometry test method with correctly positioned CCD cameras for production geometrical development of a worm gear hob with arched profile, Procedia Manufacturing, ISSN 2351-9789. 2020, Volume 51, pp.: 365-372. https://doi.org/10.1016/j.promfg.2020.10.052.,

[Á-3.] **ÁBEL, JÓZSEF**: Analysis of the mathematical correlation between post-sharped hob and worm gear tooth surface, Multidiszciplináris tudományok, 11. kötet. (2021) 4sz. pp.: 278-286. https://doi.org/10.35925/j.multi.2021.4.32

Journal article published in Hungarian language:

[Á-4.] **ÁBEL JÓZSEF**: Csigakerék fogfelület meghatározás saját fejlesztésű számítógépes programmal, Multidiszciplináris tudományok, (kiadás feladatfolyamatban) https://ojs.uni-miskolc.hu/index.php/multi/authorDashboard/submission/833)

Article published in a conference proceedings in foreign language:

[Á-5.] BALAJTI, ZSUZSA ; **ÁBEL, JÓZSEF**: Computer Aided Design of Worms in the Same Axis, Proceedings of the Thirteenth International Conference on Tools : ICT 2012 Miskolc, Hungary, University of Miskolc, (2012) pp. 317-322.

[Á-6.] BALAJTI, Z., **ÁBEL, J**.:The bijective part of the Monge cuboid for the mapping of the helix and a spatial curve arc, The 7 th International Conference Geometric and Graphics moNGeometrija 2020, 18-20 September 2020, Belgrade, Serbija.

[Á-7.] BALAJTI, ZSUZSA, **ÁBEL, JÓZSEF**: Examination for post-sharpening adjustment of cutting edge of a worm gear hob with circle arched profile in axial section, 31th International Conference on Flexible Automation and Intelligent Manufacturing FAIM2021

Article published in a conference proceedings in Hungarian language:

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