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APPLICATION OF EVOLUTIONARY METHODS IN STRUCTURE OPTIMIZATION

Booklet of PhD Theses

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1 Introduction

Optimization problems are everywhere in our daily lives. When we choose the way to go to work, when we try to pack in a backpack, or when we choose our investments to maximize the expected return, we are essentially solving an optimization problem. No formal training is required to solve a problem in natural processes, animals or plants. The rapid resolution of optimization problems for all species is key to survival, and over time, this has been favored by evolution. Obviously, these are solved heuristically rather than accurately, meaning that the approximations produced in this way are not guaranteed to be accurate. Nature tries to deduce the possible solution of a new problem from a solution of the past. This is called learning by analogy and is applied iteratively until a target state is reached or closer to the target state.

Heuristics (and metaheuristics) have been present since the formation of life on Earth, but for their scientific study had to wait until the 20^{th} century [78]. Presumably, they are so natural that it has been necessary to wait for the formal development of optimization.

To date, more than 192 procedures have been documented. In addition to the publication of new methods and procedures, the literature on setting the parameters of algorithms or increasing the accuracy of procedures has appeared and is becoming more common. The lengthy manual setting of parameters is slowly being replaced by adaptive methods [3, 17, 71, 91]. Increasing accuracy and efficiency can be enhanced by a combination of different procedures at multiple levels or in parallel [46, 47].

Almost simultaneously with the research of the methods, their applications and the research on them appeared. They can be used effectively in many areas. Without wishing to be exhaustive, some of them are highlighted: structure optimization [11, 39], shortest path problem [20], optimal location of ocean wave power plants on farms [57].

2 Review of literature

Evolutionary computation (EC) are a subset of artificial intelligence, including a subset of stochastic search procedures. These are a collection of iterative methods and procedures that use the previous results to continuously develop the possible solution, the set of solutions [19].

Their operation is mainly determined by the mathematical model that generates newer and newer solutions. The concept of their design can be several and can be grouped in several ways. Possible grouping by BURGOLYA [15]:

- *biological evolution methods*: the steps observed in evolutionary processes (e.g., selection, inheritance, etc.) are modeled using mathematical formalism. These include, but are not limited to, genetic algorithms (GA) [4, 26], genetic programming, evolutionary algorithms (EA) [19], evolutionary programming [24], and evolutionary strategies [5];
- *biological procedure, behavioral methods*: such as various swarm intelligences [2, 21, 22], species foraging methods, reproductive strategies, and many other biological behaviors [13, 28, 55, 87, 92];
- *methods using purely mathematical models*: in this case, abstract linear combination or probability modell are used [27, 71, 79].

It is important to note that evolutionary computation (EC) and evolutionary algorithms (EA) are often used interchangeably in the literature. Although EA is a subset of EC. Hereinafter, this interpretation as a synonym will be applied.

SZABÓ [80, 81] suggests a method for estimating the required number of iterations (exit condition) by analyzing the convergence and iteration history. Allowing to estimate the convergence resulting from stochastic operation, which differs from task to task.

The constantly rising prices of raw materials and energy today, the stricter environmental regulations justify and necessitate the development of optimal mechanical solutions and structures. MANKOVITS ET AL. performs rubber spring shape optimization using the finite element method in [30, 51, 52, 53]. BAKSA & PÁCZELT in [7, 8] solves an optimization problem related to contact tasks using a finite element method. There are two possible outcomes of optimization for these tasks. Optimization of kinematic quantities [60] on the one hand, and dynamic quantities [62, 63] on the other. It deals with finite element formalism of contact tasks BAKSA ET AL. in [6, 9, 65].

In connection with structure optimization, the optimal design of truss transmission towers was studied RAO in [70, 72] and SILVA ET AL. in [18]. Angular steel sections are usually used for these structures TANIWAKI [82], but due to their low deflection stiffness, it is preferable to use circular hollow sections according to ORBÁN ET AL. [58]. The transmission line is dealing with the destruction of towers RAO in [73].

Virág & Jármai in [83] reviews the optimal design of ribbed plates with different rib designs. Virág & Szirbik in [84, 85, 86] use their finite element model to study the effect of optimized ribbed plate parameter changes on eigenvalues. Assuming a uniaxial load, the eigenvalues for the loss of stability are determined by removing the ribs during the numerical test.

One of the most critical issues in the design of welded structures is stability issues. Demonstrates a method for designing box-section columns with a minimum mass Petrik et al. in [66], examining the effect of using several standard specifications and steels with different yield strengths on the minimum mass. Fire impact can be as critical a design consideration as loss of stability. Petrik et al. describes a method for the optimal design of pressure vessels under fire load in [67].

It is difficult to determine the cost functions during the optimal design in terms of structure cost, because they change over time Klansek & Kravanja in [45], Jalkanen in [32] and depend on the conditions of the given country Tímár et al. in [31]. For comparison purposes, internationally measured production times and data should be used and multiplied by a wider range of variable cost factors Jármai et al. in [35, 37, 38]. It deals with welding times and costs Pahl & Beelich in [41] and Hubka in [59]. Other costs – such as painting, cutting, plate straightening, etc. – described in Jármai in [35] and Farkas & Jármai [33, 34].

3 Applyed methods

The literary adage of there is “no free lunch,” was applied to comparing algorithms, because their outcomes are variable and difficult to compare. I strived to create uniform conditions for my simulations. The results are well observable; that the 11 chosen algorithms converge during the iterative steps with a total of 30 test functions, with a five-dimensional order of magnitude.

By converting the calculations within the iteration step of the flower pollination algorithm (FPA) to matrix and / or vector operations, and by breaking down fitness functions into hierarchical simple functions, the optimization can be run on a SIMD architecture. Compared to sequential processing, a significant increase in computational speed can be achieved, which requires parallelization of both the operations of the algorithm and the calculations associated with the fitness function.

The equations describing the cross-members of a truck's platform in terms of strength, stability and fatigue can be included in a fitness function with the help of penalty functions, for the purpose of searching for a minimum mass. Originally weight-optimized RHS crossmembers, they can be further optimized by using I-sections and reducing the number of supports.

During a cost-optimization of the main girder of a box section crane, with a firefly algorithm, it was shown that the use of steel with higher yield strength but costing more, is not recommended. The optimized cost, increases linearly as a function of payload, cubically as the span length increases, and finally according to the fatigue curve. In each case, the optimized cost functions for the different purposes are quasi-parallel to each other.

The internal forces of the truss like structures, and the stresses required for sizing, can be determined by the finite element method. By optimizing and simplifying the pre- and post-processing operations of the finite element method, I connected it with the self-adaptive differential evolution algorithm. The relationship between the two methods offers an efficient numerical calculation tool. Through the quantified example of a transmission line tower, in the case of deltoid-shaped griding, the optimal topology in terms of mass - such as the number of grids, etc. - strongly depends on the yield strength of the steel used. The use of steel with higher yield strength, does not necessarily mean a reduction in mass.

4 Goals of research

1. **goal**: For the evolutionary algorithms studied in this dissertation, such as artificial bee colony (ABC) [40, 42, 43], bee algorithm (StdBA) [68, 69], biogeography-based optimization (BBO) [10, 76, 77], differential evolution (DE) [79], adaptive differential evolution (SaDE and SaNSDE) [71, 91], firefly algorithm (FA) [87, 89], flower pollination algorithm (FPA) [88, 90], harmony search (HS) [25, 74], invasive weed optimization (IWO) [49, 56], and particle swarm optimization (PSO) [14, 75] are difficult or incompatible. incomparable performance results are reported. The test functions used to evaluate performance or the simulation environment settings are different.

My goal is to extensively simulate the listed algorithms using continuous test functions and uniform environment settings. Ranking them based on simulation results.

2. **goal**: Nowadays, the computing capacity of graphics cards can also be used for general purpose calculations [16, 29, 36]. This allows for data processing following the high computational parallel SIMD and SIMT architecture. Parallel versions of elementary, linear algebraic and numerical algorithms specifically using the CUDA API are available [44]. In some cases, they are also runtime-optimized, such as parallel reduction [54].

   My goal is to find a way to execute the FPA algorithm and the fitness function in parallel, using the possibilities provided by the graphics card. I pay special attention to objective functions that can be computed by parallel reduction. My goal is also to explore the computational capacities between normal sequential running and parallel running.

3. **goal**: Research related to optimization [1, 48, 53, 66, 84] is becoming increasingly important. Within this, the applications of evolutionary algorithms in structure optimization also play an important role [11, 38, 39, 35].

   My goal is to optimize two structures that occur in practice by evolutionary methods, such as the optimization of the crossmembers of platform of the van and the main girder of the crane. In the case of a truck platform, I examine how the optimum mass changes if we deviate from the cross-sectional geometry used in the original structure and the number of crossmembers changes. When optimizing your main crane girder, I look at the change in cost, using different hook loads, spans, and raw materials.

4. **goal**: The finite element method is an approximate calculation method based on different variational principles [12, 23, 30, 61, 64]. For truss-like structures modeled with pushed-pulled bar, this approximate calculation gives the exact solution [61].

   My goal is to find a method to combine evolutionary optimization with a finite element solution of structures that can be modeled with pulled-pushed bar elements. Solving a problem that occurs in practice with the found method. The chosen quantified engineering problem is the optimization of the lower part of a truncated pyramid-shaped transmission line tower with a truss like structure that can be modeled with rod elements. During the optimization, it is examined how the mass of the structure changes if the yield strength of the raw material is between $f_y = 235\,\mathrm{MPa}$ and $f_y = 690\,\mathrm{MPa}$, or if the number of grids changes or if it is limited the displacement of the node that originally had the largest displacement.

5 New scientific results – Theses

1st **thesis**: I have tested 11 pcs evolutionary algorithm with test function set of [50], using the same simulation environment (e.g., number of computations of objective function values, population size, etc.). The convergence of the average error values relative to the known optimum per iteration step and the distribution of the error values were illustrated, I ranked the algorithms. Based on the ranking and distribution of errors, following the "no-free lunch" and its theory, the efficiency and performance of the algorithms in solving future tasks can be estimated.

Published pulications in this topic: ⟨1⟩, ⟨9⟩, ⟨13⟩

2nd **thesis**: I proposed the parallel processing of flower pollination (FPA) algorithms and a group of fitness functions on graphics cards.

(a) For the parallel processing of a flower pollination algorithm, I propose to organize the parameters – random numbers, input and output variables – into vectors and matrices. The elements of the population organized as a matrix can be calculated independently in parallel, following the rules of SIMD and / or SIMT architecture.

(b) I have developed the decomposition of the fitness functions required to optimize *Sphere*, *Ackley's*, *Rastrigin* functions and main gridder of overhead crane into simple hierarchical functions and their processing by parallel reduction.

(c) I have shown how the dimensionless velocity increase can be achieved with the methods developed above using the parallel calculations.

Published publications in this topic: ⟨3⟩, ⟨11⟩, ⟨12⟩

3^{rd} **thesis**: Using the fitness function required to optimize the crossmember of the truck platform, and using a flower pollination algorithm (FPA), I have shown that the use of I-sections is more advantageous than the original RHS sections in terms of minimum weight, and by reducing the number of crossmembers additional weight savings can be achieved.

I developed the fitness function required for the optimization of the main girder of the cabinet section crane, which I optimized with the firefly algorithm (FA), and showed:

(a) does not make sense to use higher yield strength but more expensive steel, the cost minimum functions are quasi-parallel,
(b) as a function of the hook load, the cost function increases linearly,
(c) as a function of span, the cost function increases according to a cubic function,
(d) as a function of load cycles, the cost function follows the fatigue curve.

Published publications in this topic: ⟨2⟩, ⟨4⟩, ⟨5⟩, ⟨10⟩, ⟨14⟩, ⟨16⟩

4^{th} **thesis**: I proposed a method to generate the fitness function required for evolutionary optimization using a finite element method to optimize truss like structures to a minimum of mass with self-adaptive differential evolution. In topic of the optimization of the lower part of the transmission line tower, I showed using deltoid shaped gridding:

(a) The design of the deltoid lattice is most optimal if the point of intersection of the lattice bars forming the belt within the division is exactly half of the belt bar, otherwise the weight increase can be up to $\approx 40\%$,
(b) the use of higher yield strength steels, without changing the topology, such as the number of grid division, does not necessarily result in less weight,
(c) by limiting the displacement of the node with the largest original displacement, the optimized mass can be approximated by hyperbolas as a function of the allowed displacement,

Published publications in this topic: ⟨8⟩, ⟨15⟩, ⟨17⟩

6 Paths for further development

Given the amount of calculations to be performed; the future development direction of optimization using evolutionary algorithms is to support processing with parallel calculations.

The biggest disadvantage of the proposed method of hierarchical division of fitness functions is, that it is currently done manually. Performing preliminary manual calculations are tedious. If they're not regularly occurring, repetitive tasks, they may not even be worth it. A direction in future development research, could be to develop an algorithm to produce the necessary balanced tree structure, that will automatically generate this from the mathematical expression, along with the required input vector.

Due to the properties of the applied element model, the number of operations to be performed and the equations to be solved are relatively small in the calculations of truss like structures, using the finite element method. Their number also increase slowly or moderately as the number of elements increase. Looking at the solution of a task made up of bar elements alone, does not necessarily require a solution with parallel computation. Regarding the optimization problem where structurally the same system of algebraic equations with different coefficients must be solved thousands of times, even ten of thousands of times, a faster parallel calculation can be a legitimate prospect.

At the population level, small individual matrix operations can be combined into a large-scale task that can be efficiently processed in parallel.

7 Publications

⟨1⟩ Nagy, Sz. & Jármai, K.: Alap, hibrid és többszintű evolúciós algoritmusok, *GÉP*, 69(**2**), pp.44-51, **2018**

⟨2⟩ Nagy, Sz. & Jármai K.: Evolúciós algoritmusok és alkalmazásuk futódaru optimálásán keresztül, *Doktoranduszok fóruma 2018*, Miskolc, Hungary, **2018**

⟨3⟩ Nagy, Sz. & Jármai, K.: FPA algoritmus implementálása masszívan párhuzamos architektúrára, *GÉP*, 70(**2**), pp.16-19, **2019**

⟨4⟩ Nagy, Sz. & Jármai, K.: Futódaruhíd optimálása párhuzamos FPA algoritmussal GPU-n, *GÉP*, 71(**2**), pp.27-33, **2020**

⟨5⟩ Nagy, Sz. & Jármai, K.: Teherautó plató kereszttartójának optimálása evolúciós módszerrel, *GÉP*, 71(**3-4**), pp.86-90, 2020

⟨6⟩ Nagy, Sz. & Jármai, K. & Petrik, M. & Erdős, A. & Gafil, N. H.: Hegesztett szerkezetek tervezésének kutatása a Miskolci Egyetem, *In Gáti József. XXX. Jubileumi Nemzetközi Hegesztési Online Konferencia*, pp.1-13, **2021**

⟨7⟩ Nagy, Sz. & Jármai, K. & Petrik, M.: Miskolci Egyetem kutatási témái hegesztett szerkezetek tervezésében, *Hegesztéstechnika*, 33(**1**), pp.57-62, **2022**

⟨8⟩ Nagy, Sz & Jármai, K. & Baksa, A.: Távvezeték-torony optimálása evolúciós és VEM technikával, *GÉP*, 73(**2**), pp.17-23, **2022**

⟨9⟩ Nagy, Sz. & Jármai, K.: Application of the firefly algorithm for the optimization of cranes, *In Advances and Trends in Engineering Sciences and Technologies III.*, Tatranské Matliare, Slovakia, **2018**

⟨10⟩ Nagy, Sz. & Jármai, K.: Optimum design of overhead travelling crane, 3rd *International Conference on Engineering Sciences and Technologies: ESAT 2018*, Tatranské Matliare, Slovakia, **2018**

⟨11⟩ Nagy, Sz. & Jármai, K.: Massively parallel flower pollination algorithm, *In MultiScience - XXXIII. microCAD International Multidisciplinary Scientific Conference*, Miskolc, Hungary, **2019**

⟨12⟩ Nagy, Sz. & Jármai, K..: Reducing computation time using GPU Based Parallelization of FPA Algorithm for Optimization, 16th *Miklós Iványi International PhD & DLA Symposium*, Pécs (Online), Hungary, **2020**

⟨13⟩ Nagy, Sz. & Soltész, L.: The connection between ADT and evolutionary methods in product development, *Journal of Physics: Conference Series*, 1935(**1**), p. 012001, **2021**, doi: 10.1088/1742-6596/1935/1/012001

⟨14⟩ Nagy, Sz. & Jármai K.: GPU based parallel optimization of members of a truck floor, *Journal of Physics: Conference Series*, 1935 (**1**), p. 012004, **2021**, doi: 10.1088/1742-6596/1935/1/012004

⟨15⟩ Nagy, Sz. & Jármai, K. & Baksa, A.: Evolutionary optimization of a transmission line tower with FPA algorithm, *Design of Machine and Structures*, 11(**2**), pp.36-44, **2021**

⟨16⟩ Nagy, Sz. & Jármai, K.: Using a flower pollination algorithm to optimise the cross members of a truck floor, *The* 9th *International Conference on COMPUTATIONAL MECHANICS AND VIRTUAL ENGINEERING*, Brasov, Romania, **2021**

⟨17⟩ Nagy, Sz. & Jármai, K & Baksa, A.: Combining evolutionary optimization with finite element method for optimizing transmission-line tower, *IOP Conference Series: Materials Science and Engineering*, 1237(**1**), p. 012017, **2022**, doi: 10.1088/1757-899x/1237/1/012017

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