University of Miskolc
Faculty of Mechanical Engineering and Informatics

Stability of Shell-stiffened Circular Plates

Booklet of a PhD Theses by

Burmeister Dániel

István Sályi Doctoral School,
Basic Engineering Sciences
Research Field: Mechanics of Solids

Head of doctoral school:
Miklós Tisza
full professor

Head of research group:
Imre Kozák
professor emeritus

Supervisor:
György Szeidl
professor emeritus

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Dániel Burmeister

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1 Preliminaries

Stability investigations of engineering structures look back on a long history. Euler dealt with buckling of long slender columns. As regards the stability problems of circular plates, we mention that the first paper devoted to this question was published in 1890 [1]. Since then a number of papers have been dealt with this question. Without striving for completeness we cite some important papers [2, 3, 4, 5, 6, 7, 8, 9, 10].

There are various methods for increasing the resistance of a circular plate to buckling. We can apply diverse supports on the boundaries of the structure or at an appropriate inner location where the structure functioning is not disturbed. A possible solution is the simultaneously applied supports at the boundary and in the middle of a solid circular plate or at the inner and outer boundaries of an annular plate. Instead of a support, or in addition to the existing supports we could mount further concentric ring support(s) in order to improve stability of the plate. Additionally, if the applied support restrains rotation of the plate, the stiffening effect can be further increased.

Thevendran and Wang have investigated annular plates with torsional spring and/or simple support on their boundaries using the Rayleigh-Ritz method [11]. Choosing an appropriate spring constant they have been able to model a clamped support. Laura and his co-authors [12] have examined a circular plate which is simply supported at an intermediate radius and on its boundary together with an elastic restrain against rotation on the boundary. Under the assumption of axisymmetric deformations they have also used the Rayleigh-Ritz method.

Wang and his co-authors have investigated stability of an identical structure assuming non-axisymmetric deformations and using the Kirchhoff theory of plates [13, 14], and afterwards doing the calculations on the base of the Mindlin-Reissner plate theory [15]. Choosing the optimal placement for the support in terms of stability is also a part of their work.

Analysis of densely corrugated plate can be performed if the effect of the stiffeners are averaged or smeared out on the middle surface of the plate and an orthotropic material model is used. We can find details about the theoretical background of this technique in paper [16] by Troitsky. Simitses and his co-author [17] as well as Srinivasan and his co-author [18] have dealt with the the observance of the stiffening in this manner. In the articles cited stability investigations and vibration analysis are presented for annular and sector plates stiffened densely in radial and concentric directions. Kumelj and his co-author have investigated the stability of an annular orthotropic plate which is loaded in its own plane on the inner and outer boundaries [19].

We can apply a few discrete stiffeners to improve the stability of the plates. The stiffening effect of a curved beam attached to the boundary of a circular plate has been examined in article [20] by Phillips and his co-author. Turvey

[21] has presented experimental results for a similarly stiffened circular plate but the stability problem is left out of consideration. A further paper by Turvey and his co-authors deals with the issue of a circular plate stiffened by a single diameter stiffener, the stability problems are, however, again left out of consideration [22]. Irie and his co-authors have investigated the stability and vibration of annular plates reinforced by radial stiffeners [23].

In paper [24] by Bareeva and Lizarev the effect of red a concentric ring is introduced through the in-plane stress distribution. Rossettos and his co-author have dealt with the symmetric [25] and asymmetric buckling of ring-stiffened circular plates. The effect of the stiffener is taken into account via the flexural and torsional rigidity while the axial rigidity is ignored. The papers of Frostig and Simitses [26, 27] have not applied the simplifications of the articles cited above. Articles [24, 25, 26, 27] provide solutions both for axisymmetric and for asymmetric deformations. The concentric stiffening ring is modeled as a curved beam.

Szilassy dealt with the stability of circular and annular plates stiffened by a cylindrical shell on its outer boundary in his PhD thesis [28] and in a further article [29]. On the contrary to the problem examined by Frostig and Simitses [26, 27] he used a cylindrical shell as a stiffening element instead of a curved beam applied in papers [26, 27]. The stiffening shell is attached to the outer boundary of the circular plate – it is worthy of note, that Frostig and Simitses have not mentioned the corresponding results of Szilassy who examined two different problems. The first one is that of a circular plate, while the other one is a boundary value problem of an annular plate. It is assumed that (i) the load is an in-plane axisymmetric dead one and (ii) the deformations in the circular and annular plates and in the cylindrical shell are also axisymmetric. For solving the corresponding eigenvalue problem, he used the solution of a differential equation set up for the rotation field while the solution for the cylindrical shell is based on the theory of thin shells.

2 Objectives

After reviewing the preliminary literature the main objectives of the the present thesis are formulated in the following.

In order to expand the range of solvable eigenvalue problems in contrast to the work of Szilassy [28], [29] we shall use a differential equation set up for the deflection of the plate and not the one that is set up for the rotation field. Therefore the following objective can be composed:

Objective 1.: (a) We shall derive those nonlinear equation(s) which provide the critical load for a circular plate which is stiffened by a cylindrical shell on its boundary symmetrically to its middle plane or stiffened by a cylindrical shell only on one side of the middle plane. When deriving these equations we shall utilize the continuity conditions prescribed at the intersection line of the middle surfaces of the shell and the plate. (b) By solving the nonlinear equations obtained we shall determine the dimensionless critical load. (c) We shall clarify how the height of the shell affects the value of the critical load. (d) There arises the question if the stiffening shell could be replaced by elastic supports. If yes, what are the corresponding spring constants?

The fundamental problem in Objective 1. – how does the stiffening shell affect the critical load – is also a problem to be raised and solved for annular plates as well. Although there exists a closed form solution for the differential equation that describes the stability problem of annular plates, it is a further issue that the closed form solutions are difficult to handle, because the indexes of those Bessel functions which constitute the solution depend on the eigenvalue (the critical load) sought. This fact is also involved in

Objective 2.: Using a proper numerical procedure – it is worth testing it on the problem of the circular plate – (a) we should determine the dimensionless critical load for annular plates stiffened on its boundary symmetrically to its middle plane or stiffened only on one side of the middle plane. These investigations should be carried out for various support types. (b) On the basis of the computational results the effect of the inner radius (as an independent parameter) on the critical load should also be clarified.

Although the constant radial force system acting on the outer boundary of the structure is an axisymmetric load, there is no guarantee, that the deformation of the structure after stability loss is also axisymmetric. Consequently there arises the question whether the smallest critical load belongs to axisymmetric or to non-axisymmetric deformations. The assumption of non-axisymmetric buckling shapes supposes a more sophisticated mechanical model. These thoughts lead to the formulation of

Objective 3.: Using Fourier's method together with an appropriately choosen Galerkin function for the solution of the shell problem we shall clarify how the solution can be derived for the coefficients of the Fourier series. For the cylindrical shell we shall utilize some results of Vlasov [30] and Jezsó [31]. Furthermore the boundary and continuity conditions valid for the Fourier coefficients should also be clarified. Based on these results and by using a proper numerical algorithm the critical load should be determined for each coefficient of the Fourier series. The computations are to be carried out for various support arrangements concerning the circular/annular plates stiffened on their boundary symmetrically to the middle plane or stiffened only on one side of the middle plane.

It is a further issue how the structure behaves if the stiffening shell is attached to the plate at an intermediate radius, i.e., somewhere between the inner and outer boundaries. As regards the preliminaries we cite article [27] by Frostig. This author modeled the stiffener as a concentric ring (e.g. as a beam) which is attached at an intermediate radius to the plate. As the name implies, this assumption can be used only if the stiffener is beamlike. It is worth mentioning that there are a few publications on this topic in the scientific literature. Related to this issue our

Objective 4.: We should develop an appropriate model in order to investigate the effect of shell-stiffening at an intermediate radius. We should do calculations in order to determine (a) the critical load and (b) the optimal placement of the stiffening shell. We should make the corresponding investigations both for axisymmetric deformations and for non-axisymmetric deformations as well.

3 Performed investigations

The cross-section of the investigated structure is shown in Fig. 1. The structure consists of a circular or annular plate and a cylindrical shell attached either on the outer boundary of the plate or at an inner radius between the outer and inner boundaries of the plate. The shell is either built symmetrically to the mid-surface of the plate or – if it is applied on the boundary – attached on one side of it. The figure demonstrates a symmetric stiffening on the outer boundary of an annular plate. The structure is loaded by radial distributed forces with a constant intensity acting in the middle plane of the plate. The load is a dead one.

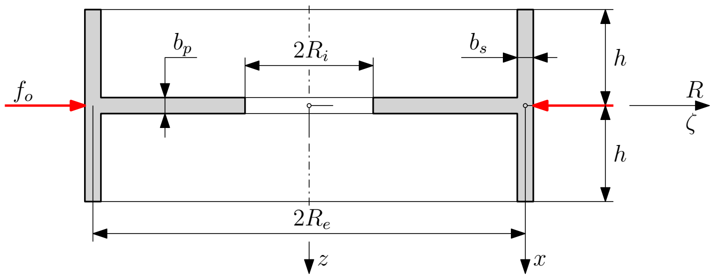

Figure 1. Geometry and load of the structure

We assume that the plate and the shell are thin, consequently we can use the Kirchhoff theory of plates and shells. It is also assumed that the problem is linear with regard to the kinematic equations and material law. Heat effects are not taken into account. The plate and the shell are made of homogeneous isotropic material.

At first we present the investigations done on stiffening on the outer boundary.

In order to solve the problem raised we separate the shell and plate from each other mentally and state the governing equations for each part. The solutions regarding the certain parts are connected through the continuity conditions.

Under the assumption of axisymmetric deformations the deflection of the cylindrical shell is obtained from the well known differential equation. The physical quantities appearing in the boundary and continuity conditions relating the shell are derived from the deflection. The actual solution for the problem is a superposition of the solutions we determine for two partial loads. In the first load the shell is subjected to the line loads coming from the load of the structure and the inner force between the elements. In the second load the shell is subjected to a distributed moment coming from the inner moment between the elements. After solving the partial problems we obtain continuity conditions between the shell and the plate.

The deformations and in-plane stress distribution in the plate originated from the in-plane load are obtained from the plane stress problem which is detailed in the literature.

Under the assumption of axisymmetric deformations we utilize the solution of the differential equation set up for the deflection together with the boundary conditions we derive a nonlinear equation which provide the critical load of symmetrically stiffened circular plate. In order to compute the roots of the nonlinear equation and the critical load a program has been written in the Fortran 90 language. The results obtained are presented in diagrams. It can be observed that the critical load increases significantly if we raise the height of the shell. The function is asymptotic, i.e. increasing the height of the shell over a certain limit has no further influence on the value of the critical load. If the plate is stiffened with a shell only on one side of it, the critical load is obtained in a bit different manner that is by examining the bounds of the deflections. In this case the we get the critical load from the condition that the displacements tend to infinity if buckling occurs.

The closed form solution of the differential equation regarding the stability problem of annular plates is difficult to handle since the index of the Bessel functions depends on the critical load. Since the index of the Bessel functions is not known, we solve the differential equation set up for the deflection of the middle surface of the plate numerically. We use the numerical solution in order to determine the critical load.

In the numerical algorithm we transform a differential equation of order four into a set of four differential equations of order one. We compute the solution of the system of equations by an appropriate numerical method (Runge-Kutta method). Substituting these solutions into the boundary conditions we get a linear homogeneous system of equation which includes the load as a parameter. The critical value of the load is obtained from the condition that the system of equations has a non-trivial solution only if the determinant of its coefficient matrix vanishes.

The algorithm has been implemented in a program code and the results we have obtained are presented in diagrams. The influence of the height of the shell and the inner radius of the plate on the critical load is shown if the stiffening is either symmetric with respect to the plate middle plain or is attached to one side of the annular plate. The effect of the height of the shell is similar to that we saw in the case of circular plates, while the critical load increases if the inner radius is getting bigger.

Under the assumption of non-axisymmetric deformations the load is axisymmetric but not the deflection. In this more general case the plane stress problem remains the same as in the former problem. If we consider the non-axisymmetric deformations of the plate and the shell, the governing equations are different and therefore the solutions of them are different as well.

If the problem is non-axisymmetric the equations are set up for two variables. In the cylindrical coordinate system we use we expand the displacement field in terms of the angle coordinate. By substituting the series in the differential equation set up for the deflection, we obtain ordinary differential equations for the members of the series. We use the solutions we have found for the calculation of the physical quantities, which are also expanded into Fourier series.

In the solution procedure of the cylindrical shell first we eliminate the shear forces from the equilibrium equations. Then we express the force resultants and moment resultants in terms of the displacement coordinates by subsequent substitutions in the equilibrium equations. This way we get the fundamental equations set up for the displacement field which constitute a system of three coupled differential equations. The fundamental equations are fullfilled identically if we calculate the displacement coordinates in terms of Galerkin functions which was introduced by Vlasov. This way we uncouple the fundamental equations. The displacement coordinates and the physical quantities in the shell derived from the displacement coordinates are all calculated in terms of the Galerkin function. The Galerkin function should be also expanded into Fourier series as we did with the displacement field of the plate. The displacement coordinates on the middle surface of the shell, the rotation field, the force resultants and moment resultants can also be expanded into Fourier series. The coefficients of these series can be calculated from the solution for the Fourier coefficients of the Galerkin function.

After clarifying the boundary and continuity conditions prescribed for the structure we obtain a homogeneous linear system of equation to determine the integration constants. The critical load belongs to the non-trivial solution of the system of equations. The computational results are presented in diagrams for circular and annular plates with stiffening symmetric to the plate or attached to one side of it. The critical loads belonging to the members of the Fourier series have been shown against the height of the shell and the inner radius of the annular plates. On the basis of the diagrams the geometric proportions can be determined for which the smallest critical load belongs to non-axisymmetric deformations.

Annular and circular plates stiffened at an intermediate radius are discussed in the following. In this case the plate has to be separated into two parts. Consequently we derive two separate solutions for the two segments of the plate in the solutions of the plane stress problem. These solutions are joined through the continuity conditions. The in-plane stress distribution in the two segments can be described with only one load parameter. As regards the solutions of the plate and the shell, these can be derived in principle identically as we have seen in the former problems.

The computational results, considering axisymmetric and non-axisymmetric deformations as well, are given in terms of the radius of the shell while the shell height has a fixed value. By analysing the diagrams we find that the smallest critical load belongs to axisymmetric deformations for circular plates. Further we can determine the geometric proportions where the smallest critical load of annular plates belongs to non-axisymmetric deformations.

4 Novel results

Novel results concern the stability problem of shell-stiffened circular or annular plates. Homogeneous isotropic material and small deformations are assumed. The shell is attached symmetrically to the middle plane of the plate or only to one side of it. Otherwise the stiffening shell might be attached to the plate either on the outer boundary or at an intermediate radius. The plane stress distribution in the plate is axisymmetric, however the deflection due to buckling can either be symmetric or non-symmetric.

According to the aforementioned objectives, the results I have attained are organized into four statements. Each statement is organized into paragraphs denoted by roman letters which are preceded by the statement number.

Statement 1. Assuming axisymmetric load and deformations

1.a. I have derived the nonlinear equation(s) that provide the critical load both for a circular plate stiffened symmetrically to its middle plane by a cylindrical shell and for a circular plate stiffened on one side of the middle plane by a cylindrical shell.

1.b. I have determined the dimensionless critical loads and the limits for the increase of the characteristic geometric sizes of the stiffener. According to the results it is worth increasing the shell height to the value $\frac{h}{R_e} = 0,3$. Then we have obtained that the critical load is 3.5–4.5 times greater for the stiffened structure if the thickness of plate and that of the shell are the same.

1.c. I have determined the spring constant of those elastic supports which could stand in for the symmetrically attached stiffening shell.

As regards the corresponding publications see references (1), (2), (3), (4) and (12).

Statement 2. Considering an annular plate stiffened on its outer boundary symmetrically to its middle plane or only on one side of the middle plane

2.a. I have developed a numerical algorithm in order to determine the dimensionless critical load.

2.b. I have clarified the effect of the inner radius on the critical load for various types of supports. I have determined numerically how the critical load of annular plates depend on the inner radius of the plate and on the height of the shell. I have also determined the limits for which it is not worth increasing the length size of the stiffener since that has no further influence on the critical load of the structure.

As regards the corresponding publications see references (2), (5) and (13).

Statement 3. Assuming axisymmetric load and non-axisymmetric deflection I have investigated the stability problem of those circular and/or annular plates which are stiffened on their outer boundary either symmetrically to the middle plane of the plate or only on one side of the middle plane.

3.a. I have derived the solution both for the plate and for the shell by expanding the displacement field of the plate together with the Galerkin function belonging to the shell into Fourier series. I have clarified the boundary- and continuity conditions that the Fourier coefficients of the physical quantities in the plate and the shell should meet.

3.b. I have determined numerically the critical loads of circular and annular plates stiffened on their outer boundary. I have determined what is the influence of the shell height and the inner radius of annular plates on the critical load of structures with uniform thickness for shell-stiffened circular and annular plates assuming simple or clamped support types on the inner boundary.

3.c According to the computational results, the smallest critical load belongs to a non-axisymmetric deflection of the annular plate provided that the plate and shell have some special dimensions – small values of $\frac{h}{R_e}$ which increases if the inner radius ρ_i gets bigger.

As regards the corresponding publications see references (6), (7), (8), (9), (14) and (15).

Statement 4. Considering a circular or annular plate with intermediate shell-stiffening

4.a. I have developed a model in order to determine the critical load of the shell-stiffened plate under the assumption of axisymmetric and non-axisymmetric deformations.

4.b. In the case of a circular plate, the smallest critical load belongs to axisymmetric deformations. An optimal placement of the shell exists at the approximate radial coordinate $\rho_s \approx 0.72$.

4.c. Considering an annular plate I have investigated the dependency on the shell radius for plates with simple or clamped support types. According to the results the smallest critical load belongs to non-axisymmetric deformations if the stiffening shell is applied near the boundaries of the plate. The smallest critical load belongs to axisymmetric deformations if the radius of the shell lies in an intermediate interval of the plate. This interval decreases with the inner radius of the plate

As regards the corresponding publications see references (1), (10), and (11).

5 Possible applications of the results

Assume that the inner space of a pressure vessel (a cylindrical shell) is separated into parts by annular (circular) plates. If the distance of the plates is above a certain limit and the buckling problem of the plates arises then the problem to be solved may coincide with the stability problem of a shell-stiffened annular plate. During the analysis we should take into account that the smallest critical load may belong to axisymmetric deformations or (under special conditions) to non-axisymmetric deformations.

The results could also be used for educational purposes, because a part of them might be worth introducing in a curriculum related to stability theory.

6 Future research

Based on the line of thought of the PhD thesis, the stability problem of a polar orthotropic circular (annular) plate stiffened by an also orthotropic cylindrical shell might be examined. The latter could cause some difficulties since serious investigations are needed to derive the solutions to be utilized from the equations of the shell theory.

A further possible question is how to calculate the critical load of the investigated structure if the load is not axisymmetric, for example: shell-stiffened circular plate loaded along its diameter.

7 Author's publications

Journal articles in foreign language

(1) Dániel Burmeister. Effects of shell-stiffening on the stability of circular plates. *Procedia Engineering*, 48:46–55, 2012.

(2) Dániel Burmeister. Stability of shell-stiffened and axisymmetrically loaded annular plates. *Technische Mechanik*, 33(1):1–18, 2013.

Journal article in Hungarian language

(3) Dániel Burmeister. Külső peremén körhengerhéjjal merevített körlemezek stabilitásvizsgálatának egyes kérdései. (Some problems of the stability analysis of circular plates stiffened by a cylindrical shell on its outer boundary) *Multidiszciplináris Tudományok: A Miskolci Egyetem közleménye*, 2(1):21–30, 2012.

Conference papers in foreign language

(4) Dániel Burmeister. Stability of a circular plate stiffened with a cylindrical shell. In *XXIV. microCAD International Scientific Conference, Section G*, pp 25–30, 2010.

(5) Dániel Burmeister. Stability of a circular plate with a hole stiffened by a cylindrical shell. In *7th International Conference of PhD Students*, pp 19–24, 2010.

(6) Dániel Burmeister. Stability of a circular plate stiffened by a cylindrical shell. In *XXV. microCAD International Scientific Conference, Section E*, pp 69–74, 2011.

(7) Dániel Burmeister. Stability of a circular plate stiffened by a cylindrical shell. In *The 4th International Conference on Computational Mechanics and Virtual Engineering – COMEC 2011*, pp 210–215, 2011.

(8) Dániel Burmeister. Stability of shell-stiffened and axisymmetrically loaded annular plates. In *7th International Conference of the Croatian Society of Mechanics (7ICCSM2012)*, 2012.

(9) Dániel Burmeister. Stability of shell-stiffened annular plates. In *The publications of the XXVI. microCAD International Scientific Conference*, 2012.

(10) Dániel Burmeister. Stability of shell-stiffened circular plates. In *The publications of the XXVII. microCAD International Scientific Conference*, 2013.

(11) Dániel Burmeister. Buckling of annular plates with intermediate shell-stiffening. In *The Publications of the MultiScience – XXVIII. microCAD International Multidisciplinary Scientific Conference*, 2014.

Conference papers in Hungarian language

(12) Dániel Burmeister. Peremén körhengerhéjjal merevített tömör körlemez stabilitása. (Stability of solid circular plates stiffened with a cylindrical shell on its boundary) In *Doktoranduszok fóruma: Gépészmérnöki és Informatikai Kar Szekciókiadványa*, pp. 40–45, 2010.

(13) Dániel Burmeister. Stability of a circular plate with a hole stiffened by a cylindrical shell - further solutions. In *Doktoranduszok fóruma: Gépészmérnöki és Informatikai Kar Szekciókiadványa*, pp. 31–36, 2011.

(14) Dániel Burmeister. Buckling of shell-stiffened and axisymmetrically loaded annular plates. In *Doktoranduszok fóruma: Gépészmérnöki és Informatikai Kar Szekciókiadványa*, pp. 18–23, 2012.

Talks in Hungarian

(15) Dániel Burmeister. Körhengerhéjjal merevített körlemez stabilitásvizsgálata nem tengelyszimmetrikus alakváltozás mellett. (Stability analysis of shell-stiffened circular plates assuming non-axisymmetric deformations) In *11th Hungarian Conference on Theoretical and Applied Mechanics, HCTAM, 2011 (XI. Magyar Mechanikai Konferencia).* Miskolci Egyetem, 2011.

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