University of Miskolc
Department of Mechanics

Lajos György Kocsán

First-order stress function-based cylindrical shell model and related *hp* dual-mixed finite elements

Booklet of PhD Theses

Supervisor: Edgár Bertóti, DSc

Miskolc-Egyetemváros
2013

Members of the Defense Committee

Chairman:

Imre Kozák, DSc	University of Miskolc

Secretary:

Ferenc Szabó, CSc	University of Miskolc

Members:

Gábor Vörös, DSc	Budapest University of Technology and Economics
István Sajtos, PhD	Budapest University of Technology and Economics
Tamás Szabó, CSc	University of Miskolc

Reviewers:

Balázs Pere, PhD	Széchenyi István University
István Hegedűs, DSc	Budapest University of Technology and Economics

1 Preliminary

1.1 Shell theories

A shell is a three-dimensional structure bounded by two curved surfaces, where the distance between these surfaces, the thickness, is small in comparison with other body dimensions. Such structures can span over relatively large areas with high strength-weight ratio, and hold applied loads in a very effective way. These advantageous properties originate from the curved geometry of the shells, which is also the main reason behind the modeling difficulties of these structures.

The most widely used shell models in the analytical and numerical computations of shells are usually derived by applying the so-called dimensional reduction procedure, where the original three-dimensional shell problem is reduced to an approximate two-dimensional one.

According to this method and adapting some hypotheses, hierarchical shell models of varying degrees of accuracy can be derived from the power series expansions of the elasticity equations of the three-dimensional shell body with respect to the thickness coordinate about the *middle surface*[1].

The well-known Naghdi shell model, the lowest-order member of the classical hierarchical shell model family, is established from the elastic three-dimensional problem due to the following hypotheses:

- the Reissner-Mindlin kinematical assumption: the displacement components are assumed to be linear functions of the thickness coordinate except the constant transverse deflection,

- the stress assumption: the normal stress in the direction normal to the shell middle surface vanishes.

In accordance with the above kinematical hypothesis the normal strain vanishes in the direction normal to the shell middle surface, i.e., there is

[1] The locus of points lying midway between the two bounding surfaces (the inner and outer, or top and bottom surfaces) is called the middle surface.

no thickness change during the deformation. This model, also referred as *shear-membrane-bending model*, includes the transverse shear.

Another shell model, which plays an important role in the analytical solutions, can be deduced by considering a third assumption, the Kirchhoff-Love kinematical hypothesis:

- The material line orthogonal to the initial middle surface remains straight, and always orthogonal to the middle surface during the deformation.

From the above assumption a significant attribute follows, namely there is no transverse shear in this model. This *membrane–bending* model is often called the Koiter shell model.

A limitation of the above mentioned classical shell models can be experienced by virtue of the modified material law, i.e., following from the Reissner–Mindlin kinematical hypothesis and the stress assumption, a modified constitutive equation – a plain stress-like material law – has to be applied. Furthermore, setting the through-the-thickness stress to be zero is restrictive in some modeling tasks.

A possible way to avoid these problems, albeit at a larger computational cost, is the application of higher-order displacement assumptions, or hierarchical displacement-based shell elements.

A *modeling error* can be used to compare dimensionally reduced shell models to the original three-dimensional model. This error occurs during the dimensional reduction procedure and depends on the middle surface geometry, the thickness and the boundary conditions. It can be said that a well-deduced shell model is *asymptotically consistent*, if the modeling error approaches zero as the thickness tends to zero.

1.2 Shell finite elements

One of the first and still used approach to the modeling of shell structures is the application of facet-shell elements, which consists in approximating the shell structure by an assemblage of planar triangular or quadrilateral elements. After a rigorous mathematical analysis, a doubt was cast as the facet-shell elements satisfy the required mathematical properties of convergence and consistency.

The Koiter shell model has a serious disadvantage in the numerical aspects of the shell modeling, namely the application of finite elements based on the Kirchhoff–Love kinematical hypothesis requires C^1 continuity of the expanded displacement components.

The shell finite elements based on the Naghdi-model is more widely used, since this model contains only first derivatives of the displacements, and therefore the related finite element formulation requires only C^0 continuity of the displacements.

Furthermore, it should be noted that the standard dimensionally reduced low-order shell finite elements suffer from several numerical difficulties. The basic source of these *numerical locking effects* arises from the small value of the thickness, i.e., when the shell becomes very thin. The standard low-order h-version triangular and quadrilateral shell elements with Naghdi-type kinematics are characterized by the most serious locking phenomena.

The most typical modeling problem of shell structures that can be experienced is the so called *boundary layer* effect. This problem originates from a typical phenomenon, namely rapidly decaying solutions of the equations of shell theory may be obtained near the boundaries. One of the effective strategies, to overcome this deterioration of the finite element solution, is the use of the local mesh refinement near the effected boundaries. A similar phenomenon called internal layers may arise due to the propagation of singularities along the asymptotic lines of the middle surface.

Other serious locking phenomena of the dimensionally reduced shell modeling are the membrane locking and the transverse shear locking. In the case of the latter locking effect, the shell finite elements appear increasingly stiffer as the thickness decreases and exhibit some parasitic shear. The more severe membrane locking occurs when the bending energy is restrained and the total strain energy is stored in membrane energy terms for small thickness values.

Several attempts have been made to design locking-free shell finite elements, and this field is still under intense research. One of them is the application of the reduced or selective integrations. Such methods are suffer from spurious zero energy modes for certain boundary conditions, however, stabilization techniques to overcome these problems have been developed. Furthermore, the use of a primal-mixed variational principle for plates and shells, which approximates the transverse shear stresses as independent variables, is also promising. There are also attempts to use mixed variational formulations based on the Hu–Washizu or the Hellinger–Reissner functional.

Using displacement-based shell models, the application of higher-order (p-version) shell finite elements is considered to be a remarkably effective strategy. In thin shell problems, however, the membrane locking can only partially be avoided by using p-type approximation: although these elements are verified to be locking-free in the energy norm computations, in the stress computations numerical locking can be observed. To improve the performance of the hp shell elements, an adaptive technique is also under research.

In the finite element modeling of shell structures, the application of the so-called degenerated shell elements derived from the three-dimensional continuum elements by carrying out the integrations numerically with respect to the thickness coordinate are also in use. These finite elements suffer from several additional numerical locking effects, beside the numerical problems that occur in the dimensionally reduced shell finite elements.

Finally, we note that the application of dimensionally reduced shell models – instead of three-dimensional models – is rewarding, since the discretization has to be done only over the middle surface and no discretization across the thickness is needed. Thus, the use of a shell model reduces the computational effort and guarantee more reliable solutions for thin structures.

1.3 Dual-mixed finite elements

Dual-mixed variational formulation and finite element models have several advantages. In these complementary energy-based mixed finite element formulations the stresses, the variables of primary interest in many engineering applications, are directly approximated, i.e., they provide better convergence rates and higher accuracy for the stresses than strain energy-based primal-mixed and the conventional displacement-based formulations. Furthermore, as the inverse constitutive law is used, these finite elements are free from the volumetric-locking effects.

The dual-mixed principles can be deduced from the complementary energy variational principle, where the only variable is the self-equilibrated stress field, i.e., a stress field that satisfies the translational and the rotational equilibrium equations. The stress boundary condition is also a subsidiary condition of this principle. Such a stress field can be obtained by the introduction of the second-order stress function tensor. The second-order compatibility equations, compatibility boundary conditions and displacement boundary conditions are weakly satisfied.

Using these approaches, Fraeijs de Veubeke proposed a finite element procedure: the so-called equilibrium method. Later, it was analyzed numerically for two-dimensional elasticity problems. Since these methods use second-order stress functions, the C^1 continuous approximation is required, which makes it rather difficult and complicated to establish a numerically efficient and well manageable equilibrium method and stress elements for general problems.

If the rotational equilibrium equation is satisfied in a weak form, the first-order stress functions should be applied. This leads to the Fraeijs de Veubeke variational principle in terms of stresses and (infinitesimal) rotations, where the rotations play the role of the Lagrangian multipliers. The finite element methods deduced in this way require only C^0 continuity of the stress functions.

The analogy between the weak forms of the stress- and rotation-based formulation and the displacement-pressure formulation of elasticity (and the velocity-pressure formulation of Stokes flow) has been pointed out. Furthermore, locking-free dual-mixed hp-finite element models for plane elasticity and plate problems using first-order stress functions and rotations have been developed.

Applying the classical kinematical hypotheses, stress- and rotation-based finite elements for plates and shells have also been developed.

We also mention that a two-field dual-mixed variational principle can be obtained, if the translation equilibrium is incorporated into the complementary energy functional using the Lagrangian multiplier technique. Hence a Hellinger–Reissner type variational principle in terms of a priori symmetric stresses can be obtained, where the displacement vector and the stress tensor are the directly approximated variables. However, it is not an easy task, to create stable finite element spaces when the two-field dual-mixed Hellinger–Reissner type variational principles using a priori symmetric stresses are considered.

This problem can be circumvented by imposing the symmetry condition on the stress tensor in a weak sense. Thus the obtained three-field dual-mixed Hellinger–Reissner-type variational principle enforces both the translational and rotational equilibrium equations in a weak sense. The development of related finite elements is based on the independent approximations of the displacements, non-symmetric stresses and rotations.

1.4 Stress-based shell theories and finite elements

The inspiration of this work comes from a different research direction: the stress-based shell theories. A common and important property of these dimensionally reduced shell models is that the classical kinematical hypotheses are not applied in the construction of these shell models.

In the works of Kozák, the theory of a general shell model in terms of the symmetric stresses has been developed. Subsequently, a dimensionally reduced cylindrical shell model was derived in terms of the self-equilibrated stress field, which is obtained with the aid of the second-order stress function tensor.

One difficulty of the numerical formulation based on the second-order stress functions lies in the fact that the C^1 continuity of the tractions has to be guaranteed. To avoid this problem the symmetry restriction on the stress tensor can be released and the Fraeijs de Veubeke variational principle can be applied.

Using this principle, first-order stress function-based plate models has been developed by Bertóti and found to be promising in locking-free finite element computations.

Due to the use of non *a priori* symmetric stresses, the stress components can be approximated in a different order across the thickness. The latter property also ensures more freedom in the selection of the stress approximation across the thickness.

In view of this shell models based on the non a priori symmetric stresses can be introduced with the aid of the corresponding equilibrium hypothesis, namely the equilibrium equations regarding to the stresses omitted by the truncation of the stress power series[2]. is considered to be identically satisfied.

2 Objectives

The objective of the dissertation is to present a new dimensionally reduced cylindrical shell model and the related *hp* finite element formulations on the basis of the two-field dual-mixed Fraeijs de Veubeke variational principle. In this work, we combine the effectiveness of the *hp*-version finite elements with the stress-based shell theories.

The main properties of the developed dimensionally reduced cylindrical shell model are the following:

- the approximation order of all the variables with respect to the thickness coordinate is determined by the *equilibrium hypothesis*,

- the classical assumptions – the Kirchhoff-Love and the Reissner-Mindlin kinematical hypotheses as well as the corresponding stress hypothesis – are not applied,

[2]The expanded stress components that are adequately small and treated to be zero

- three-dimensional (inverse) constitutive equations can be used without any modifications,
- the fundamental variables are the first-order stress functions and the rotations,
- the symmetry of the stress tensor is satisfied in an integral average sense.

2.1 The Fraeijs de Veubeke variational principle

The functional of the two-field dual-mixed Fraeijs de Veubeke variational principle, on which the new shell model is based, can be deduced from the complementary energy functional by adding a Lagrange multiplier term which ensures the symmetry of the stress tensor:

$$\mathcal{F}(\sigma^{pq}, \varphi^c) = \int_V \left[\overset{*}{W}(\sigma^{pq}) + \in_{qpc} \varphi^c \sigma^{pq} \right] \mathrm{d}V - \int_{S_u} \tilde{u}_p \sigma^{pq} n_q \, \mathrm{d}S, \tag{1}$$

where σ^{pq}, φ, $\tilde{u}$, $\in_{qpc}$ and n_q are, respectively, the stress tensor, the (infinitesimal) rotation vector, the prescribed displacement, the covariant permutation tensor and the outward normal of the bounding surface of the three-dimensional elastic body. The complementary strain energy density is given by

$$\overset{*}{W}(\sigma^{pq}) = \frac{1}{2} \sigma^{k\ell} \varepsilon_{k\ell}(\sigma^{pq}) = \frac{1}{2} \sigma^{k\ell} D_{k\ell pq} \sigma^{pq}, \tag{2}$$

where the symmetric strain tensor ε_{pq} can be expressed by the aid of the fourth-order elastic compliance tensor $D_{k\ell pq}$.

Applicability of (1) requires that the stress tensor σ^{pq} satisfies *a priori* the translational equilibrium equations

$$\sigma^{k\ell}_{..;\ell} + q^k = 0 \qquad \text{in } V \tag{3}$$

and the stress boundary conditions $\sigma^{k\ell} n_\ell = \tilde{p}^k$ on S_p, where q^k is the density of the body forces and $\tilde{p}^k$ is the prescribed surface tractions on the surface S_p. The displacement boundary condition on surface S_u is imposed weakly in (1).

A self-equilibrated stress field, i.e., a stress field that fulfills (3), can be obtained by introducing the first-order stress function tensor $\Psi^k_{.p}$, as

$$\sigma^{k\ell} = \in^{\ell pq} \Psi^k_{.q;p} + \hat{\sigma}^{k\ell} \quad \text{in } V, \tag{4}$$

where $\hat{\sigma}^{k\ell}$ is a particular solution to (3).

2.2 Geometric description

In the Cartesian frame of reference x^a, we consider a three-dimensional cylindrical shell with curvilinear coordinate system ξ^k. The parametric form of the middle surface is defined by the set of equations $x^1 = \xi^1$, $x^2 = R\sin\xi^2$ and $x^3 = R\cos\xi^2$, where R is the radius of the middle surface. Let L denote the length and d be the uniform thickness of the shell. Then it occupies the region

$$V = \left\{ \mathbf{r}(\xi^k) | 0 < \xi^1 < L, 0 \le \xi^2 < 2\pi, -\frac{d}{2} < \zeta < \frac{d}{2} \right\}, \tag{5}$$

bounded by the lateral surfaces

$$S^{0\vee L} = \left\{ \mathbf{r}(\xi^k) \,|\, \xi^1 = 0 \vee L\,,\, 0 \le \xi^2 < 2\pi\,,\, -\frac{d}{2} \le \zeta \le \frac{d}{2} \right\}, \tag{6}$$

and the inner and outer surfaces

$$S^{\pm} = \left\{ \mathbf{r}(\xi^k) \,|\, 0 < \xi^1 < L\,,\, 0 \le \xi^2 < 2\pi\,,\, \zeta = \pm\frac{d}{2} \right\}. \tag{7}$$

2.3 The cylindrical shell model

In the development of the new dimensionally reduced cylindrical shell model, all the variables are expanded into power series with respect to the thickness coordinate around the shell middle surface[3].

[3]For example, the stress tensor is expanded as $\sigma^{ab}(\xi^a) = \sum_{i=0}^{\infty} {}_i\sigma^{ab}\,(\xi^\alpha)\,(\xi^3)^i$.

The translational equilibrium equations for cylindrical shells in terms of the expanded stresses can be given in the following forms:

$$ {}_{i}\sigma^{1\beta}_{..,\beta} + \frac{1}{R}\,{}_{i}\sigma^{13} + (i+1)\ {}_{i+1}\sigma^{13} + \ {}_{i}q^{1} = 0, \tag{8} $$

$$ {}_{i}\sigma^{2\beta}_{..,\beta} + \frac{1}{R}\left({}_{i}\sigma^{32} + \ {}_{i}\sigma^{23}\right) + (i+1)\ {}_{i+1}\sigma^{23} + \ {}_{i}q^{2} = 0, \tag{9} $$

$$ {}_{i}\sigma^{3\beta}_{..,\beta} - R\,{}_{i}\sigma^{22} + \frac{1}{R}\,{}_{i}\sigma^{33} + (i+1)\ {}_{i+1}\sigma^{33} + \ {}_{i}q^{3} = 0, \tag{10} $$

where $i = 0, 1, 2, ..., \infty$. From (8–10) it follows that the stresses σ^{a3} have to be approximated by one degree higher than $\sigma^{a\beta}$ to satisfy the i-th expanded equilibrium equations. This is admissible, as the symmetry of the stress tensor is weakly imposed in the applied variational principle. In the dissertation we apply the approximations

$$ \begin{aligned} \sigma^{a\beta}(\xi^{a}) &= {}_{0}\sigma^{a\beta}(\xi^{\alpha}) + \ {}_{1}\sigma^{a\beta}(\xi^{\alpha})\,\xi^{3}, && (11)\\ \sigma^{a3}(\xi^{a}) &= {}_{0}\sigma^{a\beta}(\xi^{\alpha}) + \ {}_{1}\sigma^{a\beta}(\xi^{\alpha})\,\xi^{3} + \ {}_{2}\sigma^{a\beta}(\xi^{\alpha})\,(\xi^{3})^{2} && (12) \end{aligned} $$

for the stresses.

The application of this approximation requires an *equilibrium hypothesis*, such that the expanded stresses ${}_{i}\sigma^{a\alpha}$ and ${}_{j}\sigma^{a3}$ regarding to $i > N$ and $j > N+1$ are adequately small and the equilibrium equations corresponding to the i-th power of the thickness coordinate are identically satisfied, where $N \geq 1$. Applying higher-order approximations of the stresses ($N \geq 2$), it is possible to introduce higher-order (hierarchic) shell models.

In the presented shell model the classical kinematical hypotheses related to the approximation of the displacement field are not applied. The approximation of other quantities (the rotation vector for example) with respect to the thickness coordinate is a corollary of the equilibrium hypothesis. Furthermore, another property distinguishing our shell model from the classical shell models is that the structure of the stress (or strain) tensor makes it possible to apply unmodified three-dimensional constitutive equations.

After a variable-number reduction procedure, which can be achieved by incorporating the loads prescribed on the inner and outer surfaces and a priori satisfying some stress-symmetry conditions, a general cylindrical shell model and, after further simplifications, a cylindrical shell model for axisymmetric problems is developed.

Carrying out some mathematical manipulations, the fundamental differential equation of the axisymmetric shell problem for one of the stress function unknowns can be derived. Then the analytical solutions of the developed shell model and that of the Koiter shell model is compared indicating that the new shell model is asymptotically consistent.

2.4 Dual-mixed *hp* finite elements for cylindrical shells

The finite element formulation of the new dual-mixed axisymmetric cylindrical shell model has been developed. With the aid of two benchmark problems, the following was proved numerically: the rates of h- and p-convergences are high for both thin ($d/R = 10^{-2}$) and very thin ($d/R = 10^{-4}$) shells. It was also demonstrated that convergence rates using p-refinement are much better than that of with h-refinement.

The finite element formulation of the dual-mixed general cylindrical shell model, capable of both h- an p-type approximations, has been also developed. Two mixed finite element spaces were tested, both of them yielded promising results in the performed computational tests. The obtained convergence curves of relative errors in energy norm and in maximum norm of the stresses are indicated that the proposed finite element spaces provide good convergence results for both thin ($d/R = 10^{-2}$) and extremely thin ($d/R = 10^{-6}$) shells.

Acknowledgement: I acknowledge the financial support received from the TÁMOP-4.2.1.B-10/2/KONV-2010-0001 project and from the TÁMOP-4.2.2/B-10/1-2010-0008 project in the framework of the New Hungarian Development Plan. The realization of these projects is supported by the European Union, co-financed by the European Social Fund. I also acknowledge the support provided by the Hungarian Scientific Research Fund under Grant No. OTKA T49427.

3 Theses

Thesis 1

Starting from the three-dimensional Fraeijs de Veubeke dual-mixed variational principle, I have derived a new dimensionally reduced shell model for linear elasticity problems of cylindrical shells. In this shell model the fundamental (independent) variables are the first-order stress functions and the rotations. The approximation of the stresses with respect to the thickness coordinate follows from the equilibrium hypothesis, while the approximation of other quantities is a corollary of the stress approximation and the structure of the applied variational principle. I have developed the cylindrical shell model without the application of the classical kinematical hypotheses and, therefore, the shell model uses unmodified three-dimensional (inverse) constitutive equations.
▶ Related publications: [6,9,13,14].

Thesis 2

I have derived the equation system of the axisymmetric elastostatic problems of cylindrical shells in terms of first-order stress functions and rotations as independent variables. Applying the variable-number reduction procedure and neglecting the independent torsion problem, the number of the independent variables has been reduced to four: to three stress function and one rotation components. I have derived the fundamental differential equation of the axisymmetric shell problem for one of the stress function unknowns. I have derived the analytical solution of this two-point boundary value problem, which served as a reference solution in the numerical tests of the new shell model and the related finite elements. I have demonstrated the asymptotically consistent behavior of the new shell model.
▶ Related publications: [1,2,3,4,8,10,11,15,16,17,20,21,22].

Thesis 3

I have developed a new dual-mixed *hp* finite element model for the axisymmetric problems of cylindrical shells. I have introduced a polynomial interpolation space for the approximated variables – the three first-order stress functions and one rotation – and determined the zero-energy mode of the stress functions for the shell element. Investigating two representative model problems, I have tested and demonstrated the computational performance and the numerical efficiency of the new *hp* finite element model. I have proven through these computations that the *hp* cylindrical shell elements developed for axisymmetric problems are robust and reliable, i.e., the convergence rates in the energy norm as well in the stresses are free from the negative effects of the decreasing thickness of the shell for either h- or p-refinement.
► Related publications: [1,2,4,7,15,16,19,18].

Thesis 4

I have made developments towards a new dual-mixed *hp* finite element model for general problems of cylindrical shells. I have investigated the approximation capabilities and the numerical efficiency of different finite element polynomial spaces for the stress function and rotation variables. The proper polynomial approximation spaces have been selected by performing numerical tests on special model problems. I have proven through these numerical investigations that the approximation capabilities of the new cylindrical shell finite element spaces are independent of the thickness of the shell and the results shows good convergence rates in the energy norm and in the stress computations for either thin or extremely thin shells.
► Related publications: [5,12,13,14].

Theses accepted by the Defense Committee: 1,2,3

4 Author's publications

The following publications were made in the topic of the dissertation:

Articles in journals

[1] Tóth, B. and Kocsán, L. G. Comparison of dual-mixed h- and p-version finite element models for axisymmetric problems of cylindrical shells. *Finite Elements in Analysis and Design,* **65**:50–62, 2013. *(IF*=1.058)*

[2] Kocsán, L. G. Derivation of a dual-mixed *hp* finite element model for axisymmetrically loaded cylindrical shells. *Archive of Applied Mechanics*, **81**:1953–1971, 2011. *(IF=0.950)*

[3] Kocsán, L. G. Forgásszimmetrikusan terhelt körhengerhéj analitikus megoldása elsőrendű feszültségfüggvényekkel. (Analytical solution for axisymmetrically loaded cylindrical shells using first-order stress functions.) *GÉP*, **LX**(6):21–31, 2009.

Conference papers

[4] Kocsán, L. G. and Tóth, B. A note on dual-mixed cylindrical shell models. In *Proceedings of XXVI. microCAD International Scientific Conference*, pages 1–6, University of Miskolc, Hungary, March 29–30, 2012.

[5] Kocsán, L. G. Finite Element Formulation of a Cylindrical Shell Model in Terms of First-Order Stress Functions and Rotations. In *Proceedings XXV. of microCAD'2011 International Scientific Conference*, pages 19–24, University of Miskolc, Hungary, Section E: Applied Mechanics, March 31 – April 1, 2011.

[6] Kocsán, L. G. On the selection of first-order stress functions for cylindrical shells. In *Proceedings of XXIV. microCAD International Scientific Conference*, pages 49–54, University of Miskolc, Hungary, Section G: Applied Mechanics, March 18–19, 2010.

[7] Kocsán, L. G. Finite element model for cylindrical shells using first-order stress functions. In *Proceedings of XXIII. microCAD International Scientific Conference*, pages 13–18, University of Miskolc, Hungary, Section F: Applied Mechanics, March 19–20, 2009.

[8] Kocsán, L. G. Fraeijs de Veubeke-féle variációs elv alkalmazása körhengerhéjakra (Applying the Fraeijs de Veubeke variational principle in case of cylindrical shells). In *Proceedings of Applied Mechanical Research Mini-symposium*, pages 1–6, Széchenyi István University, Hungary, November 12, 2008.

[9] Kocsán, L. G. Zero Energy Modes in a Two-Field Dual-Mixed Shell Model Using Stress Functions. In *Proceedings of microCAD'2008 International Scientific Conference*, pages 31–37, University of Miskolc, Hungary, Section F: Applied Mechanics, March 20–21, 2008.

[10] Kocsán, L. G. On the first order stress function approach for cylindrical shells. In *Proceedings of PhD Students' Forum*, pages 57–63, University of Miskolc, Hungary, November 13, 2007.

[11] Kocsán, L. G. Linear Shell Model using the Stress and Rotation Field. In *Proceedings of microCAD'2007 International Scientific Conference*, pages 31–36, University of Miskolc, Hungary, Section E: Applied Mechanics, March 22–23, 2007.

Conference presentations

[12] Kocsán, L. G. Two-field dual-mixed *hp* finite element method for cylindrical shells. *The 5th International Conference on Modelling of Mechanical and Mechatronics Systems*, Zemplínska Šírava, Slovakia, November 6–8, 2012.

[13] Kocsán, L. G. *hp* Finite Element Model for Cylindrical Shells Using First-Order Stress Functions. *38th Solid Mechanics Conference*, Warsawa, Poland, August 27–31, 2012.

[14] Kocsán, L. G. A Fraeijs de Veubeke-féle variációs elven alapuló végeselem-modell körhengerhéj feladatokra (Fraeijs de Veubeke variational principle-based finite element model for cylindrical shells). *XI. Magyar Mechanikai Konferencia*, University of Miskolc, Hungary, August 29–31, 2011.

[15] Kocsán L. G. and Bertóti E., A two-field dual-mixed *hp* Finite Element Model for cylindrical shells. *Workshop on Higher Order Finite Element and Isogeometric Methods*, Cracow, June 27–29, 2011.

[16] Kocsán, L. G. and Bertóti, E. Cylindrical shell model in terms of first-order stress functions and rotations, *37th Solid Mechanics Conference*, Warsaw, Poland, September 6–10, 2010.

[17] Kocsán L. G. Kétmezős lineáris körhengerhéj-modell elsőrendű feszültségfüggvényekkel (Two-field linear shell model using first-order stress functions). *X. Magyar Mechanikai Konferencia*, University of Miskolc, Hungary, August 27–29, 2007.

Talks

[18] Kocsán, L. G. Fraeijs de Veubeke-féle variációs elv alkalmazása körhengerhéjakra (Applying the Fraeijs de Veubeke variational principle in case of cylindrical shells), *Doktoranduszok Szemináriuma (PhD Students' Seminar)*, University of Miskolc, Hungary, April 6, 2009.

[19] Kocsán, L. G. Finite element model for cylindrical shells using first-order stress functions. *Otto von Guericke University, Magdeburg*, Institute of Mechanics. Magdeburg, Germany, December 18, 2008.

[20] Kocsán, L. G. Többmezős duál variációs elvek a lineáris rugalmasságtanban (Multi-field dual-mixed variational principles in linear elasticity). *Doktoranduszok Szemináriuma (PhD Students' Seminar)*, University of Miskolc, Hungary, May 26, 2008.

[21] Kocsán, L. G. Linear shell model using the stress and rotation field. *Otto von Guericke University, Magdeburg*, Institute of Mechanics. Magdeburg, Germany, December 17, 2007.

[22] Kocsán, L. G. Lineáris körhengerhéj-modell feszültségmezővel és forgásmezővel (Linear shell model using the stress and rotation field). *Doktoranduszok Szemináriuma (PhD Students' Seminar)*, University of Miskolc, Hungary, November 11, 2007.

5. Magyar nyelvű összefoglaló

A vékonyfalú héjak, héjszerkezetek ipari alkalmazása rendkívül széleskörű. Ennek oka abban keresendő, hogy a héjszerkezetek méretükhöz képest hatékonyan képesek nagy terheléseket hordani, miközben rendkívül jó tömeg-teherbírás aránnyal rendelkeznek. E hasznos tulajdonságok a héjak görbült geometriájából, illetve a héj vékonyságából fakadnak, amelyek egyben az analitikus, illetve a numerikus számítási nehézségek okozói is.

A klasszikus héjmodellek felépítésekor a héj vékonyságát kihasználó kinematikai hipotéziseket alkalmazunk, ezek segítségével redukálva az eredeti háromdimenziós feladatot kétdimenziós feladattá. A dimenzió szerinti redukció egyszerűbbé teszi az analitikus megoldások előállítását, illetve javítja a vékony szerkezetek numerikus modellezésének hatékonyságát és megbízhatóságát.

A klasszikus héjmodelleknél alkalmazott, az elmozdulásmező közelítésére vonatkozó hipotézisek hátrányos következményeként, az eredeti háromdimenziós anyagegyenletek csak módosításokkal vehetők figyelembe, valamint a vastagság irányában ébredő normálfeszültség elhanyagolása nehézséget okozhat egyes modellezési feladatoknál.

A jelenleg elterjedt héj-végeselemek nem minden esetben szolgáltatnak megbízható eredményeket, különös tekintettel a mérnöki szempontból elsőrendű fontosságú feszültségmezőre.

Az értekezés egy új, a klasszikus megközelítéstől eltérő, feszültségfüggvényeken alapuló körhengerhéj-modell és *hp*-verziós körhengerhéj-végeselemek kifejlesztésének irányába tett lépéseket foglalja magába.

A disszertációban ismertetett héjmodell a Fraeijs de Veubeke-féle variációs elvből kiindulva került levezetésre, amely elvnek a nem *a priori* szimmetrikus feszültségmező és a forgásmező az alapváltozói. Az elv mellékfeltételei a transzlációs egyensúlyi egyenletek és a feszültségi peremfeltételek. A feszültségmezőre vonatkozó transzlációs egyensúlyi egyenleteket – a disszertációban szereplő héjmodell esetében – az elsőrendű feszültségfüggvények bevezetésével elégítjük ki.

A kifejlesztett körhengerhéj-modell főbb jellemzői a következők:

- a változók vastagság menti közelítése az egyensúlyi hipotézisből (*equilibrium hypothesis*) következik,
- a klasszikus Kirchhoff-Love- és Reissner-Mindlin-féle kinematikai hipotézisek és a vonatkozó feszültségi hipotézis nélkül történt a héjmodell felépítése,
- a háromdimenziós (inverz) anyagegyenletek módosítások nélkül alkalmazhatók,
- a modell alapváltozói az elsőrendű feszültségfüggvény tenzor és a forgásvektor,
- a feszültségek szimmetriája az alkalmazott variációs elv segítségével integrál értelemben teljesülnek.

A disszertációban megtalálhatóak – általános és forgásszimmetrikus terhelési esetre – az elsőrendű feszültségfüggvények segítségével felépített körhengerhéj-modell és a vonatkozó héj-végeselemek. Levezetésre került a héjmodell analitikus megoldása forgásszimmetrikus esetre, amely visszavezethető egy állandó együtthatós negyedrendű differenciálegyenlet megoldására. A kapott eredmények alapján igazolható, hogy az új körhengerhéj-modell aszimptotikusan korrekt.

Az egyváltozós (forgásszimmetrikus) és a kétváltozós (általános) terhelési esetre h-, illetve p-verziós végeselem-modellek kerültek kifejlesztésre a disszertációban levezetett új körhengerhéj-modell alapján. A bevezetett héj-végeselemek segítségével számítható numerikus eredményeket a fent említett analitikus megoldással vetettem össze.

A jelen kutatás keretében elvégzett vizsgálatok azt mutatják, hogy az új héjelemek nem érzékenyek a héj vastagságának csökkentésére és vékony héjaknál is numerikus konvergencia problémák nélküli megoldásokat adnak a mérnöki szempontból fontos feszültségmezőre nézve, mind h-, mind p-típusú approximáció esetén.